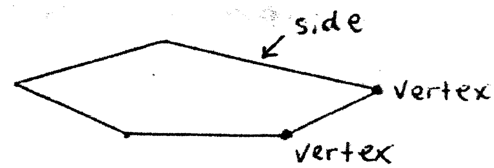
6.1 – Polygons

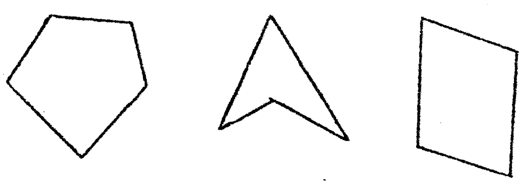
1. **Polygon** – a plane figure that meets the following conditions:

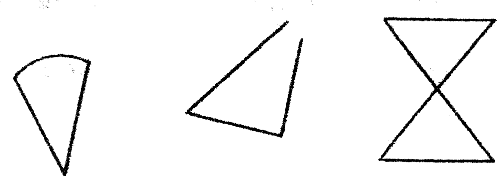
    1) It is formed by three or more segments called **sides,** such that no two sides with a common endpoint are collinear.
    2) Each side intersects exactly two other sides, one at each endpoint.

2. **Vertex** – each endpoint of a side. *(plural – vertices)*



Examples: Polygons Not Polygons





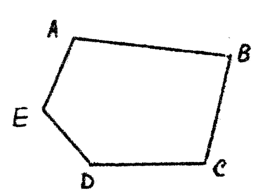
3. Polygons are named by the number of sides they have.

Number of Sides	Type of Polygon		Number of Sides	Type of Polygon
3	Triangle		8	Octagon
4	Quadrilateral		9	Nonagon
5	Pentagon		10	Decagon
6	Hexagon		12	Dodecagon
7	Heptagon		*n*	*n*-gon

4. A polygon can be named by listing its vertices *consecutively.*

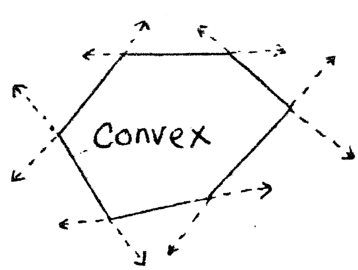
ABCDE or DEABC

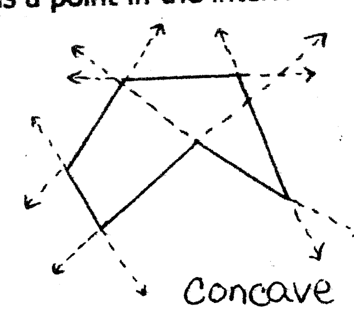
Not CEBDA



5. A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon.

6. A polygon that is not convex is **concave.**



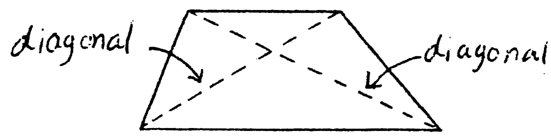


7. A polygon is **equilateral** if all of its sides are congruent.

8. A polygon in **equiangular** if all of its interior angles are congruent.

9. A polygon is **regular** if it is equilateral and equiangular.

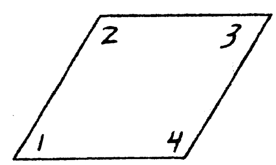
10. A **diagonal** of a polygon is a segment that joins two *nonconsecutive* vertices.



Theorem 6.1 – Interior Angles of a Quadrilateral

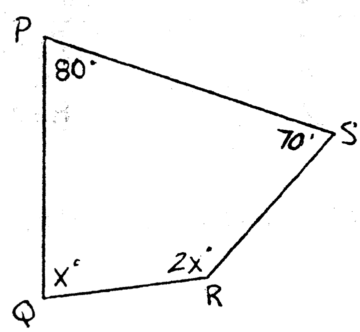
The sum of the measures of the interior angles of a quadrilateral is $360°$.

$m\angle 1 + m\angle 2 + m\angle 3 + \angle 4 = 360°$



Like triangles, quadrilaterals have both *interior* and *exterior* angles. If you draw a diagonal in a quadrilateral, you divide it into two triangles, each of which has interior angles with measures totaling $180°$. So, a quadrilateral is $2(180°)$ or $360°$.

<u>Example</u> :



Find $m\angle Q$ and $m\angle R$.

$x° + 2x° + 70° + 80° = 360°$

$3x + 150 = 360$

$3x = 210$

$x = 70$

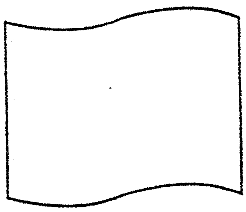
$m\angle Q = x° = 70°$

$m\angle R = 2x° = 140°$

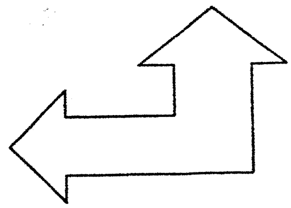
Section 6.1 Class Examples

Decide whether the figure is a polygon. If not, explain why.

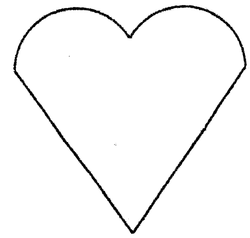
1.



2.

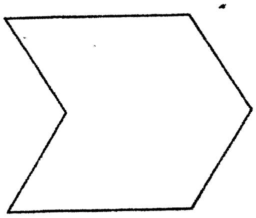


3.

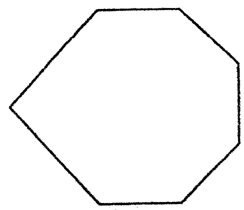


Use the number of sides to tell what kind of polygon the shape is. Then state whether the polygon is *convex* or *concave*.

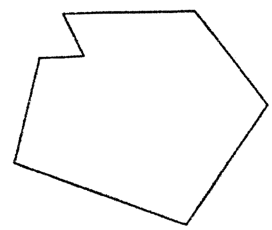
4.



5.

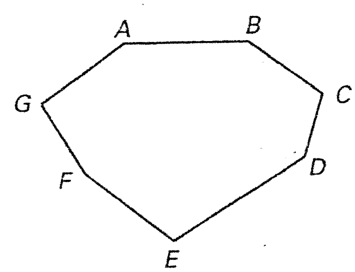


6.

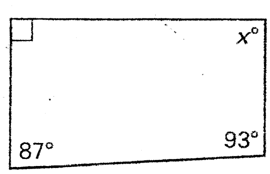


Use the diagram at the right to answer the following.

7. Name the polygon by the number of sides it has.
8. Polygon $ABCDEFG$ is one name for the polygon. State two other names.
9. Name all of the diagonals that have vertex E as an endpoint.
10. Name the nonconsecutive angles to $\angle A$.



11.



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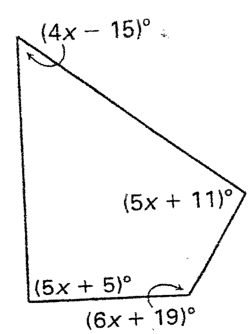
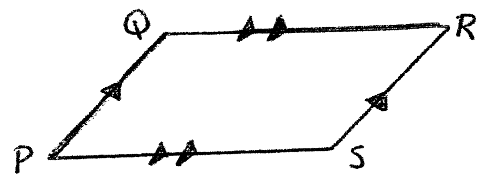


6.2 – Properties of Parallelograms

1. **Parallelogram** – a quadrilateral with both pairs of opposite sides parallel.



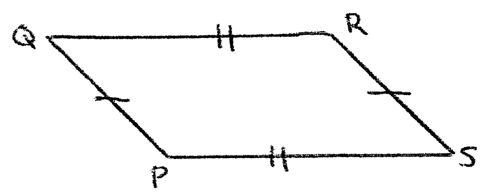
parallelogram PQRS

symbol: ▱ PQRS

Theorem 6.2

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

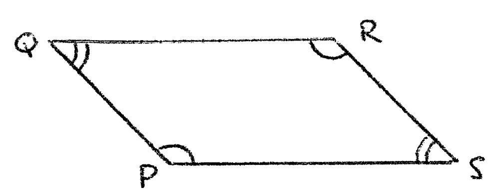
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$



Theorem 6.3

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

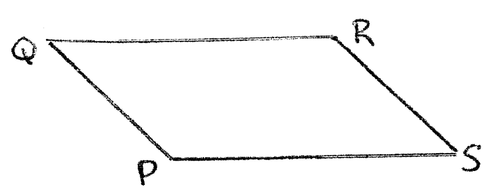
$\angle P \cong \angle R$ and $\angle Q \cong \angle S$



Theorem 6.4

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

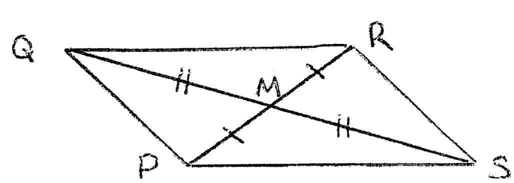
$m\angle P + m\angle Q = 180°$
$m\angle Q + m\angle R = 180°$
$m\angle R + m\angle S = 180°$
$m\angle S + m\angle P = 180°$



Theorem 6.5

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



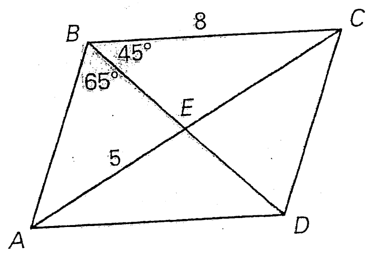
Section 6-2 Class Examples

Using Properties of Parallelograms

ABCD is a parallelogram. Find the lengths and angle measures.

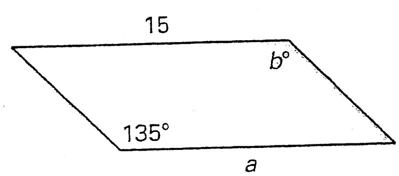
a. AD **b.** EC

c. $m\angle ADC$ **d.** $m\angle BCD$

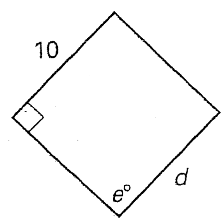


Find the value of each variable in the parallelogram.

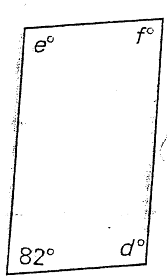
1.



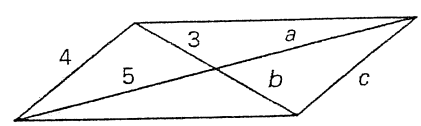
2.



3.

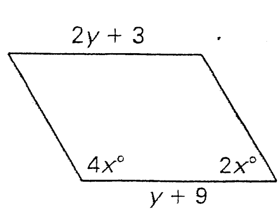


4.



Find the value of each variable in the parallelogram.

7.



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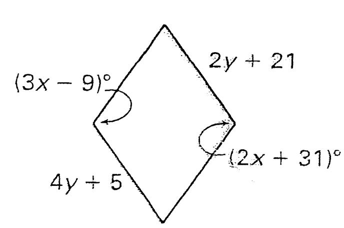
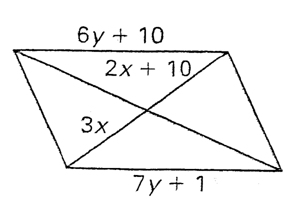


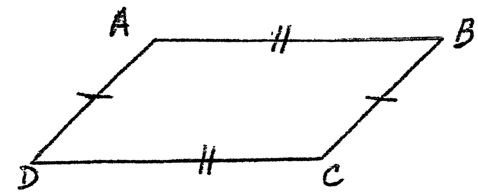
9.



6.3 – Proving Quadrilaterals are Parallelograms

Theorem 6.6

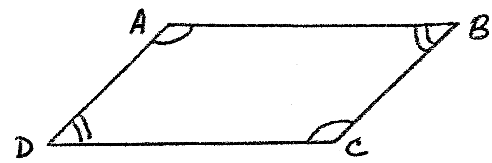
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram

Theorem 6.7

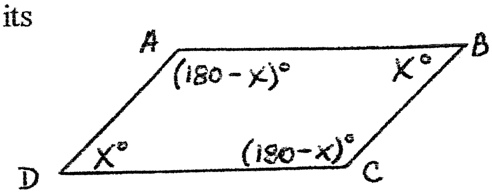
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram

Theorem 6.8

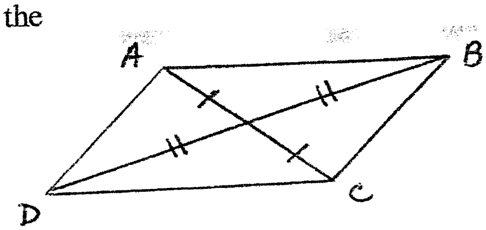
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



ABCD is a parallelogram

Theorem 6.9

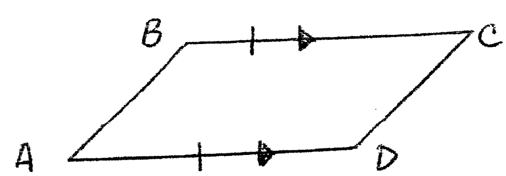
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



ABCD is a parallelogram

Theorem 6.10

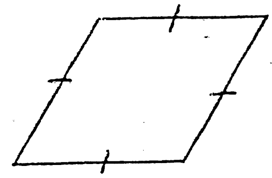
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



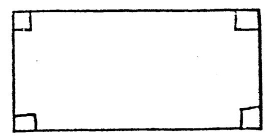
ABCD is a parallelogram

6.4 – Rhombuses, Rectangles, and Squares

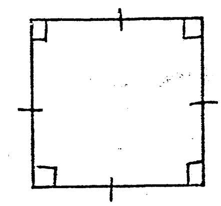
1. **Rhombus** – parallelogram with four congruent sides.



2. **Rectangle** – parallelogram with four right angles.



3. **Square** – parallelogram with four congruent sides and four right angles.



Rhombus Corollary – A quadrilateral is a rhombus if and only if it has four congruent sides.

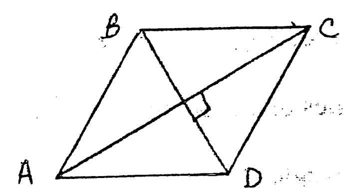
Rectangle Corollary – A quadrilateral is a rectangle if and only if it has four right angles.

Square Corollary – A quadrilateral is a square if and only if it is a rhombus and a rectangle.

Theorem 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

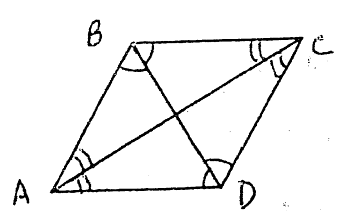
ABCD is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



Theorem 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

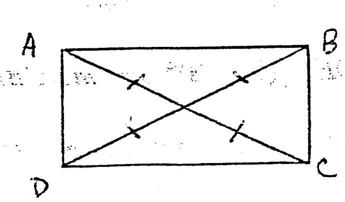
ABCD is a rhombus if and only if
$\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$ and
$\overline{BD}$ bisects $\angle ADC$ and $\angle CBA$.



Theorem 6.13

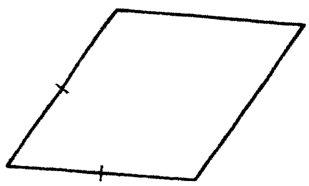
A parallelogram is a rectangle if and only if its diagonals are congruent.

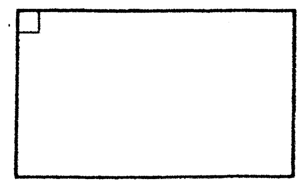
ABCD is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

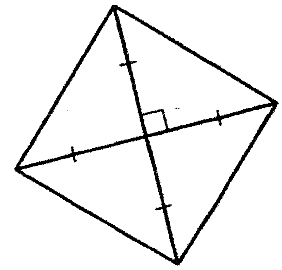


Geometry w/Applications   Practice 6.4 (A)

Each figure is a parallelogram. Identify the special type.

1. 

2. 

3. 

Name **all** of the types of quadrilaterals which have the following properties.
Parallelogram, Rectangle, Rhombus, or Square.

4. The diagonals congruent. ____________________

5. All sides congruent.____________________

6. Both pairs of opposite sides are congruent.____________________

Find the length or angle measurement. (Hint: Draw each figure)

7. *WXYZ* is a square.

$WX = 1 - 10x$

$YZ = 14 + 3x$

$XY = \underline{\ ?\ }$

8. *WXYZ* is a rhombus.

$m\angle X = 24(10 - x)°$

$m\angle Z = 6(x + 15)°$

$m\angle Y = \underline{\ ?°\ }$

In the diagram shown, MATH is a rhombus with m∠MAT = 58°.

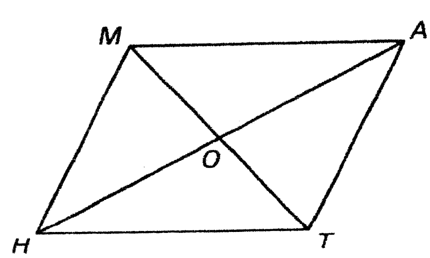
9. m∠MOH

10. m∠MHO

11. m∠HTM

12. MO =

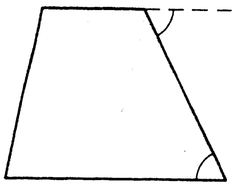
13. MA =



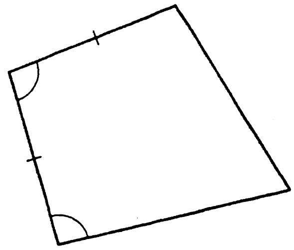
Practice 6.5

Decide whether the figure is a trapezoid. If it is, is it an isosceles trapezoid?

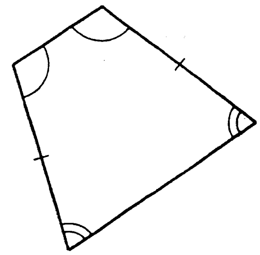
1.



2.

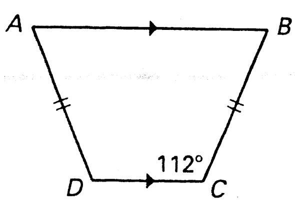


3.



Find the angle measures of *ABCD*.

4.



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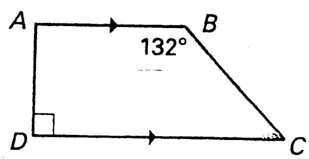
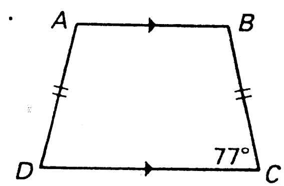
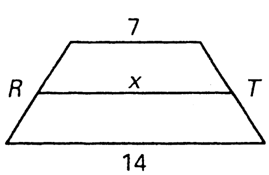


6.

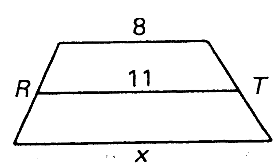


The midsegment of the trapezoid is $\overline{RT}$. Find the value of *x*.

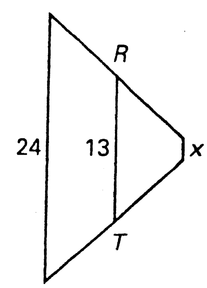
7.



8.

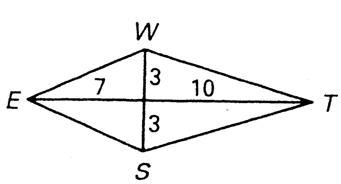


9.

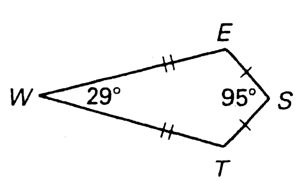


Find the length of the sides to the nearest hundredth, or the measure of the angles in kite *WEST*.

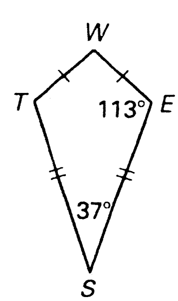
10.



11.

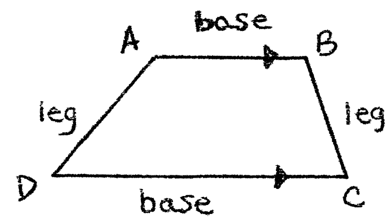


12.



6.5 – Trapezoids and Kites

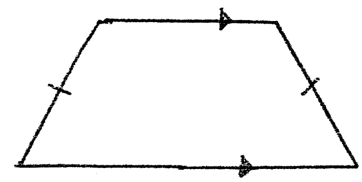
1. **Trapezoid** – a quadrilateral with exactly one pair of parallel sides.



2. **Bases** – the parallel sides of a trapezoid.

3. **Legs** – the nonparallel sides of a trapezoid.

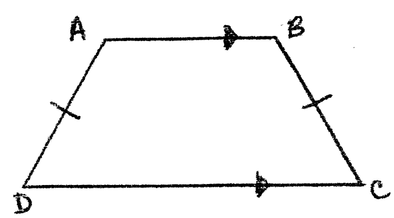
4. **Isosceles Trapezoid** – a trapezoid with congruent legs.



Theorem 6.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

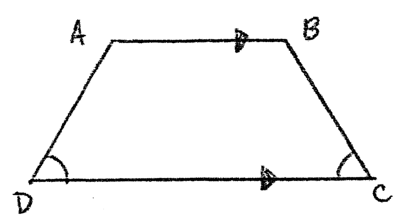
$\angle A \cong \angle B$ and $\angle C \cong \angle D$



Theorem 6.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

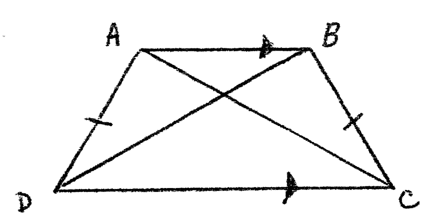
ABCD is an isosceles trapezoid.



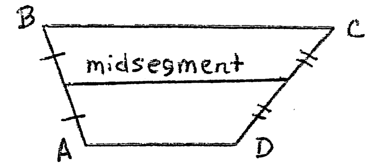
Theorem 6.16

A trapezoid is isosceles if and only if its diagonals are congruent.

ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



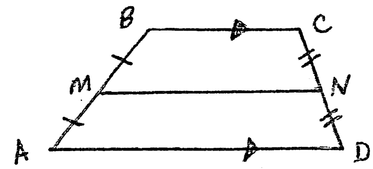
5. **Midsegment** of a trapezoid – segment that connects the midpoints of the legs.



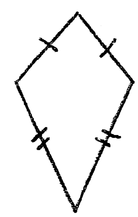
Theorem 6.17 **Midsegment Theorem for Trapezoids**

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

$\overline{MN} \parallel \overline{AD}$, $\overline{MN} \parallel \overline{BC}$, $\overline{MN} = ½ (AD + BC)$

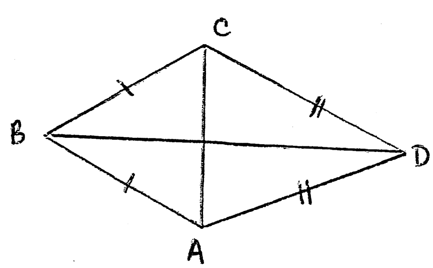


6. **Kite** – a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



Theorem 6.18

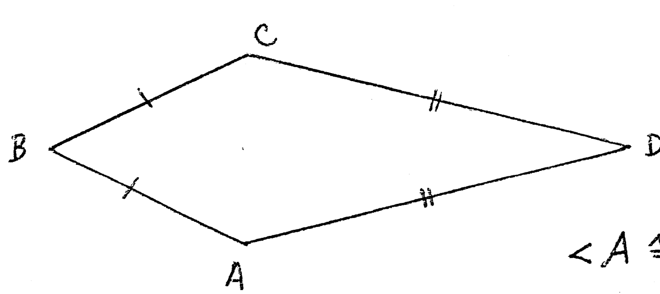
If a quadrilateral is a kite, then its diagonals are perpendicular.



$\overline{AC} \perp \overline{BD}$

Theorem 6.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.



$\angle A \cong \angle C$

$\angle B \ncong \angle D$

6.6 Notes

slope =

length =

2 matching pair – parallelogram, rhombus, rectangle, square

1 matching pair – trapezoid, isosceles trapezoid

0 matching pair – kite

*** If non-parallel sides are opposite reciprocals – rectangle, square**

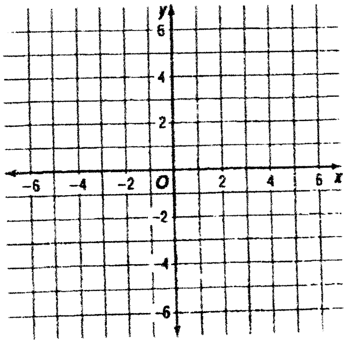
Find the length of non-parallel sides

same – rhombus, square, isosceles trapezoid

different – parallelogram, rectangle, trapezoid

Examples: Given the coordinates of *P*, *Q*, *R*, and *S*, what kind of quadrilateral is *PQRS*?

1. $P(2, 5)$ $Q(-2, 3)$ $R(2, 1)$ $S(6, 3)$



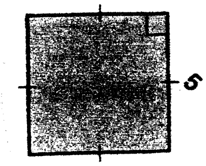
2. $P(-5, -2)$ $Q(1, -2)$ $R(3, 0)$ $S(1, 4)$

3. $P(1, 7)$ $Q(5, 9)$ $R(8, 3)$ $S(4, 1)$

Section 6.7 Areas of Triangles and Quadrilaterals

Area of a Square *Postulate*

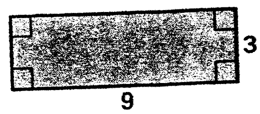
$A = s^2$



Area Addition Postulate : The area of a region is the sum of the areas of its nonoverlapping parts.

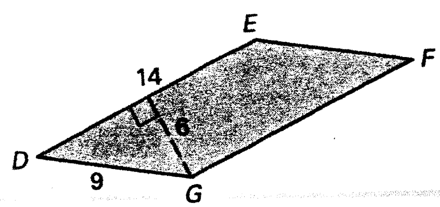
Area of a Rectangle Theorem

$A = bh\ (lw)$



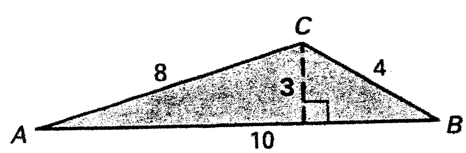
Area of a Parallelogram Theorem

$A = bh$



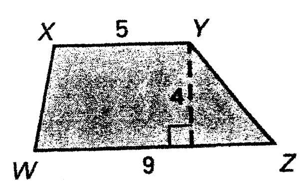
Area of a Triangle Theorem

$A = \frac{1}{2}bh$



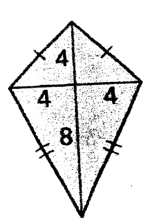
Area of a Trapezoid Theorem

$A = \frac{1}{2}h(b_1 + b_2)$



Area of a Kite Theorem

$A = \frac{1}{2}d_1d_2$



Area of a Rhombus Theorem

$A = \frac{1}{2}d_1d_2$