Chapter 8 – Similarity
8.1 – Ratio and Proportion

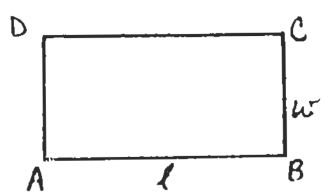
1. If a and b are two quantities measured in the ***same*** units, then the **ratio** of a to b is $\frac{a}{b}$ with $b \neq 0$.

- The ratio of a to b can also be written as $a : b$.
- Ratios are usually expressed in simplified form (lowest terms).

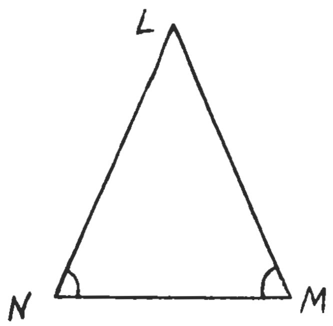
Example: Simplify the ratios.

a) $\frac{12cm}{24cm}$ b) $\frac{30\,\text{min}}{45\,\text{min}}$ c) $\frac{12cm}{4m}$ d) $\frac{6ft}{18in}$

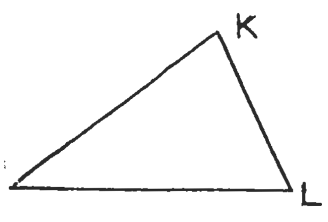
Example: The perimeter of rectangle ABCD is 60 centimeters. The ratio of AB : BC is 3 : 2. Find the length and width of the rectangle.



Example: The perimeter of the isosceles triangle shown is 56 inches. The ratio of LM : MN is 5 : 4. Find the lengths of the sides and the base of the triangle.



Example: The measure of the angles of ΔJKL are in the *extended ratio* of 1 : 2 : 3. Find the measures of the angles.



<u>8.2</u> – Problem Solving in Geometry with Proportion

1. Additional Properties of Proportions

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

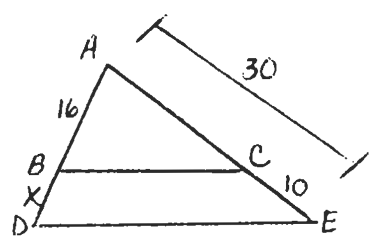
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

<u>Example</u>: Tell whether the statement is true.

a) If $\frac{p}{6} = \frac{r}{10}$, then $\frac{p}{r} = \frac{3}{5}$.

b) If $\frac{3}{b} = \frac{d}{7}$, then $\frac{3+b}{d} = \frac{d+7}{b}$.

<u>Example</u>: In the diagram $\frac{AB}{BD} = \frac{AC}{CE}$, find the length of $\overline{BD}$.



2. The **geometric mean** of two positive numbers a and b is the positive number x such that

$$\frac{a}{x} = \frac{x}{b}$$

If you solve this proportion for x, you find that $x = \sqrt{a \cdot b}$, which is a positive number.

<u>Example</u>: Find the geometric mean of the following sets of numbers.

a) 8 and 18

b) 36 and 128

c) 3 and 12

2. **Proportion** – an equation that equates two ratios. $\frac{a}{b} = \frac{c}{d}$

- The numbers a and d are the **extremes** of the proportion.
- The numbers b and c are the **means** of the proportion.

3. **Properties of Proportions**

   1. **Cross Product Property** – the product of the extremes equals the product of the means.

      If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

   2. **Reciprocal Property** – If two ratios are equal, then their reciprocals are also equal.

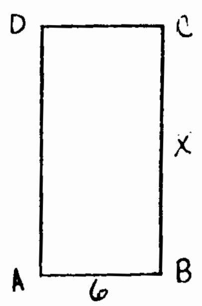
      If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

Example: Solve the proportions.

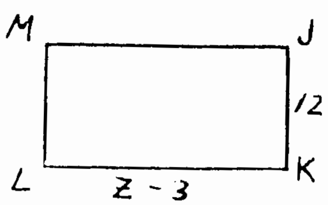
a) $\frac{4}{x} = \frac{5}{7}$

b) $\frac{3}{y+2} = \frac{2}{y}$

Example: Using rectangle ABCD, solve for the variable. The ratio of AB : BC is 3 : 8.



Example: Using rectangle JKLM, solve for the variable. The ratio of JK : KL is 2 : 3.



Class Examples 8.2

Complete the sentence

1. If $\frac{a}{b}=\frac{5}{6}$, then $\frac{b}{a}=$

2. $\frac{a}{b}=\frac{5}{6}$, then $\frac{a}{5}=$

Decide whether the statement is true or false.

3. $\frac{c}{d}=\frac{3}{4}$, then $\frac{c-d}{d}=\frac{-1}{4}$

4. $\frac{c}{d}=\frac{5}{6}$, then $\frac{5}{d}=\frac{c}{6}$

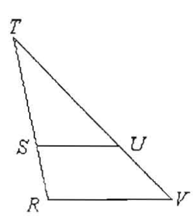
Find the geometric mean of the two numbers.

5. 4 and 16

6. 7 and 14

Use the diagram and the given information to find the unknown length.

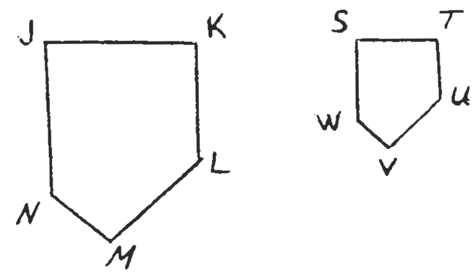
7. Given $\frac{TS}{TR}=\frac{TU}{TV}$. Find SR if TS = 12, TU = 16, UV = 5



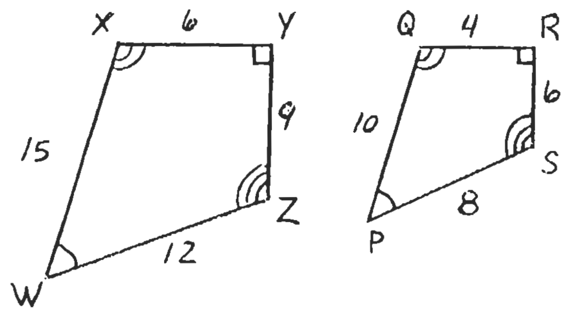
8.3 – Similar Polygons

1. **Similar Polygons** – two polygons such that their corresponding angles are congruent and the lengths of corresponding sides are proportional.

Example: Pentagons JKLMN and STUVW are similar. List all pairs of congruent angles. Write the ratios of the corresponding sides in a statement of proportionality.



Example: Decide whether the figures are similar. If they are similar, write a similarity statement.

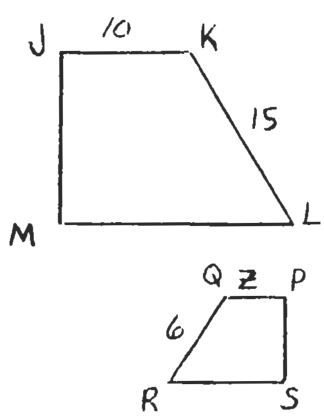


2. **Scale Factor ($a : b$)** – the ratio of the lengths of two corresponding sides of two similar polygons.

3. **Theorem 8.1** – If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If KLMN ~ PQRS, then $\dfrac{KL+LM+MN+NK}{PQ+QR+RS+SP}=\dfrac{KL}{PQ}=\dfrac{LM}{QR}=\dfrac{MN}{RS}=\dfrac{NK}{SP}$.

Example: Quadrilateral JKLM is similar to quadrilateral PQRS. Find the value of z.



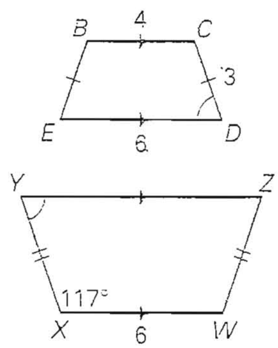
Class Examples 8.3

List all pairs of congruent angles and write the statement of proportionality.

1. $\Delta HOT \sim \Delta NDA$

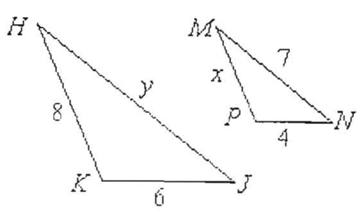
2. $\Delta MPR \sim \Delta SPX$

In the diagram quadrilateral *BCDE* ~ quadrilateral *WXYZ*.



4. Find the scale factor of quadrilateral $BCDE$ to quadrilateral $WXYZ$.
5. Find the scale factor of quadrilateral $WXYZ$ to quadrilateral $BCDE$.
6. Find the length of $\overline{XY}$.
7. Find the measure of $\angle D$.
8. Find the perimeter of quadrilateral $WXYZ$.
9. Find the ratio of the perimeter of $WXYZ$ to the perimeter of $BCDE$.

10 Find the value of x and y if $\Delta HKJ \sim \Delta MPN$

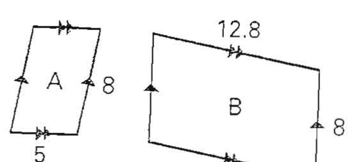


11. The ratio of one side of ΔHOT to the corresponding side of ΔNDA is 6:7.

If the perimeter of ΔNDA is 98 cm, what is the perimeter of ΔHOT ?

Decide whether the polygons are similar. If so, find the scale factor of Figure A to Figure B.

12.



8.4 - Similar Triangles

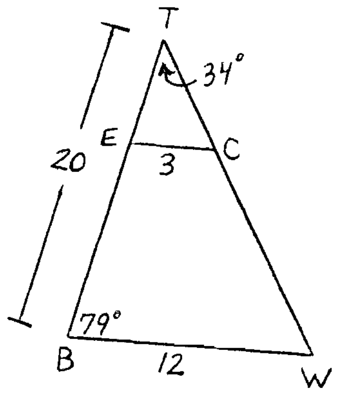
Example 1:

In the diagram, $\Delta BTW \sim \Delta ETC$.

a) Write the statement of proportionality for the similar triangles.

b) Find $m\angle TEC$.

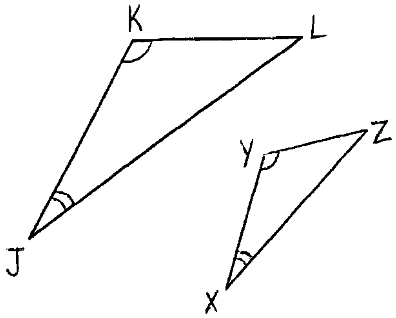
c) Find ET and BE.



Postulate 25: Angle-Angle (AA) Similarity Postulate

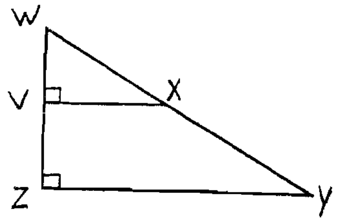
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle K \cong \angle Y$ and $\angle J \cong \angle X$, then $\Delta JKL \sim \Delta XYZ$.



Example 2:

Explain why $\Delta WVX \sim \Delta WZY$.

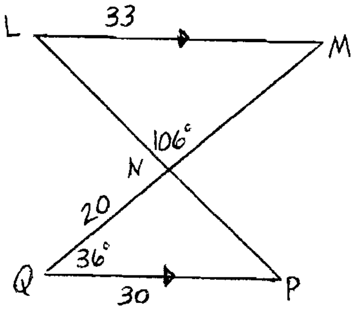


Example 3:

In the diagram, $\Delta LMN \sim \Delta PQN$.

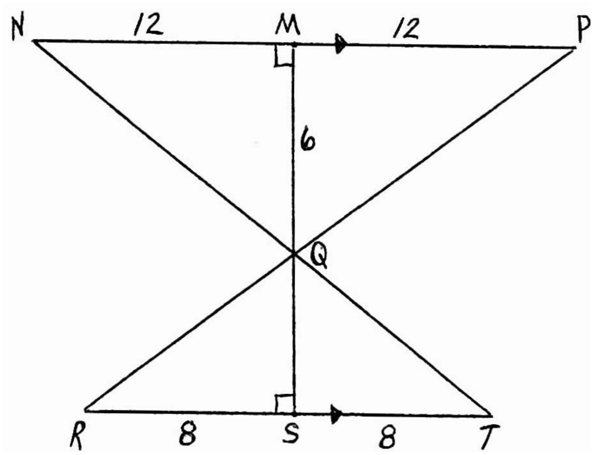
a) Find $m\angle M$ and $m\angle P$.

b) Find MN and QM.



Example 4:

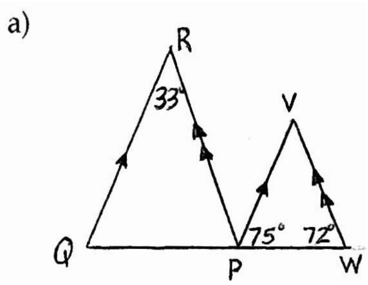
Find the length of altitude $\overline{QS}$.



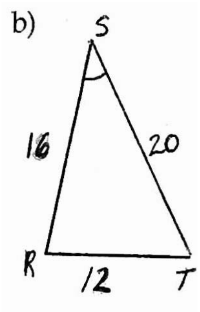
Example 5:

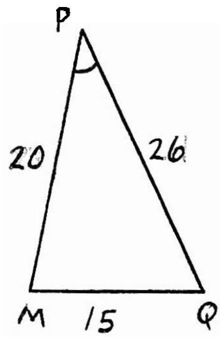
Determine whether the triangles can be proved similar. If so, write a similarity statement. If they are not similar, explain why.

a)



b)

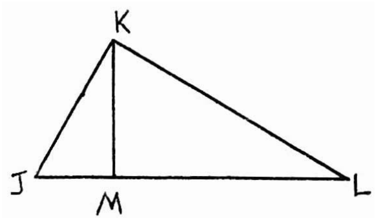




Example 6:

Fill in the paragraph proof.

Given: $\overline{KM} \perp \overline{JL}$ and $\overline{JK} \perp \overline{KL}$
Prove: $\Delta JKL \sim \Delta JMK$



Given ______________ and ______________, $\angle JMK$ and $\angle JKL$ are ____________ angles.

Since all _____________ angles are ______________, $\angle JMK$ ______ $\angle JKL$. By the

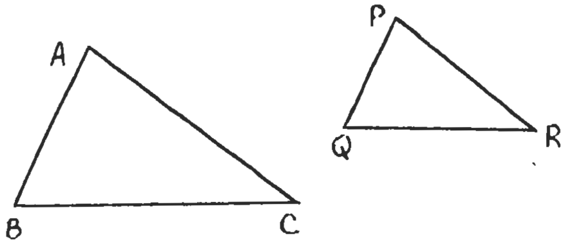
______________ property, $\angle J \cong \angle J$. So, $\Delta JKL \sim \Delta JMK$ by the ________ Similarity Postulate.

8.5 – Proving Triangles are Similar

Theorem 8.2 – Side-Side-Side (SSS) Similarity Theorem

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

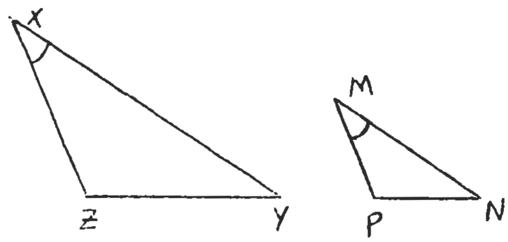
If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$, then $\Delta ABC \sim \Delta PQR$.



Theorem 8.3 – Side-Angle-Side (SAS) Similarity Theorem

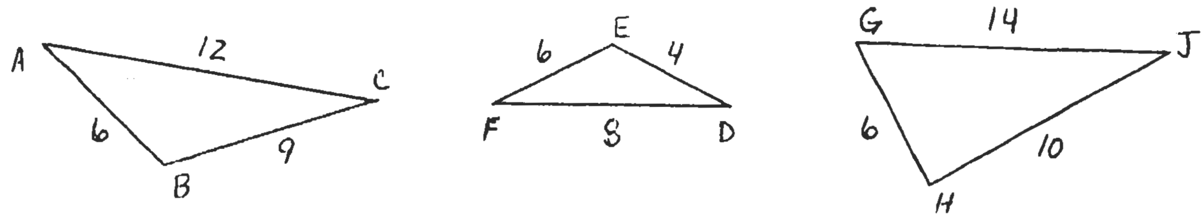
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\Delta XYZ \sim \Delta MNP$.



Example 1

a) Which of the following three triangles are similar?



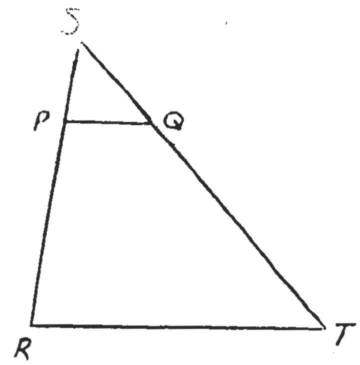
b) Write the similarity statement(s) for the similar triangles.

c) Find the scale factor(s) for the similar triangles.

Example 2 Write a paragraph proof.

Given: SP = 4, PR = 12, SQ = 5, QT = 15

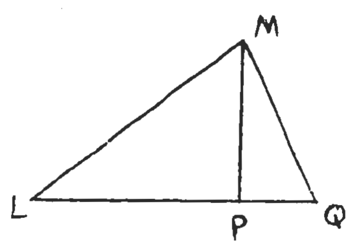
Prove: $\Delta RST \sim \Delta PSQ$



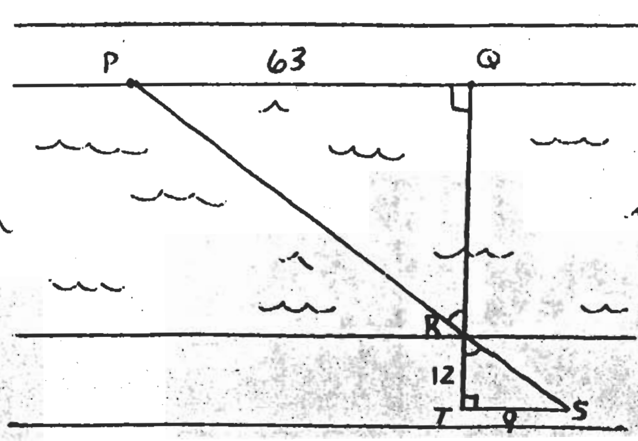
Example 3 Write a paragraph proof.

Given: $\angle L \cong \angle PMQ$ and $MP \perp LQ$

Prove: $\Delta LMP \sim \Delta MPQ$



Example 4 To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find RQ.

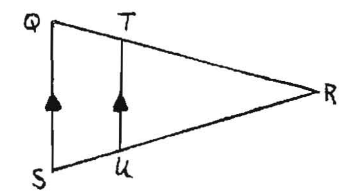


8.6 – Proportions and Similar Triangles

Theorem 8.4 – Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

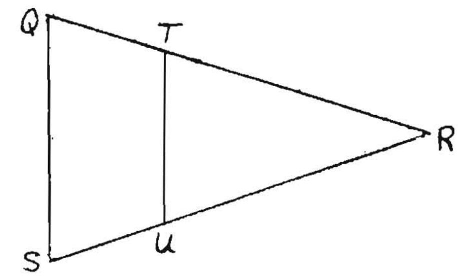
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.



Theorem 8.5 – Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

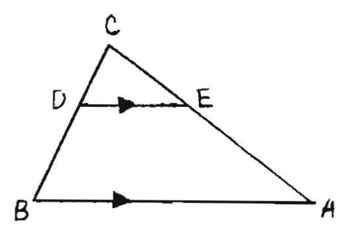
If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.



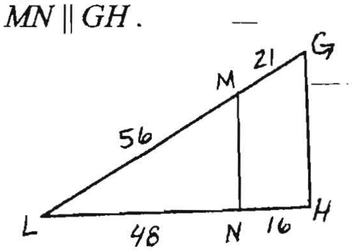
Example 1

In the diagram $\overline{AB} \parallel \overline{ED}$, BD = 8, DC = 4, and AE = 12

What is the length of $\overline{EC}$?



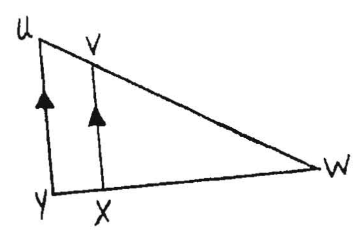
Example 2 Given the diagram, determine whether $\overline{MN} \parallel \overline{GH}$.



Example 3

In the diagram $\overline{UY} \parallel \overline{VX}$, UV = 3, UW = 18, and XW = 16.

What is the length of $\overline{YX}$?

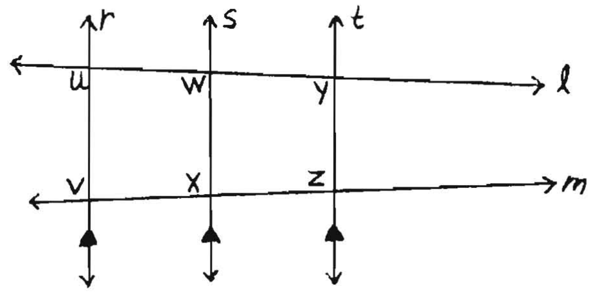


Theorem 8.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

If $r \parallel s$ and $s \parallel t$, and ℓ and m intersect r, s, and t,

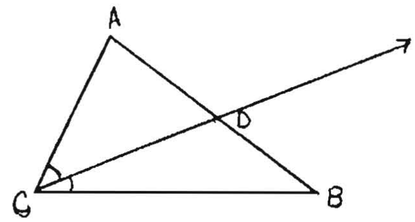
then $\frac{UW}{WY} = \frac{VX}{XZ}$.



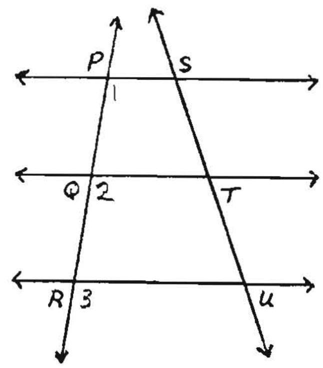
Theorem 8.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

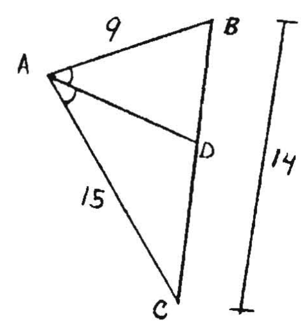
If $\overrightarrow{CD}$ bisects $\angle ACB$, then $\frac{AD}{DB} = \frac{CA}{CB}$.



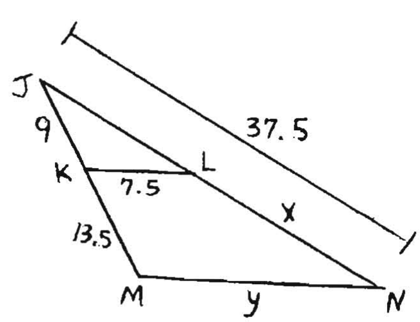
Example 4 In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$, and PQ = 9, QR = 15, and ST = 11. What is the length of $\overline{TU}$?



Example 5 In the diagram, $\angle CAD \cong \angle DAB$. Use the given side lengths to find the length of $\overline{DC}$.



Example 6 In the diagram $\overline{KL} \parallel \overline{MN}$. Find the values of the variables.

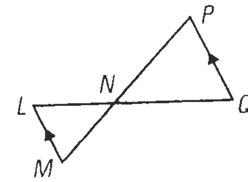


Homework Answers: pages 483 to 485

$\angle L \cong \angle Q, \angle M \cong \angle P, and\ \angle N \cong \angle N$

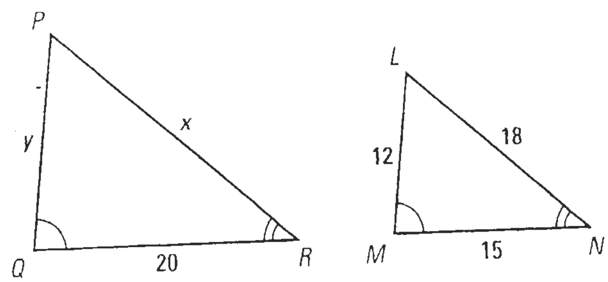
11. $\dfrac{LM}{QP} = \dfrac{MN}{PN} = \dfrac{LN}{QN}$

11.



12. ΔLMN

13. $\dfrac{PQ}{LM} = \dfrac{QR}{MN} = \dfrac{RP}{NL}$



14. $\dfrac{20}{15} = \dfrac{y}{12}$

15. $\dfrac{15}{20} = \dfrac{18}{x}$

16. y = 16

17. x = 24

18. No m∠C = 31 and m∠D = 47°.
Angles are not congruent.

19. yes: ΔPQR ~ ΔWPV

20. No: $\dfrac{RS}{MP} \neq \dfrac{ST}{PQ}$

21. yes; ΔXYZ~ ΔGFH

22. No m$\angle E$ = 94°, $\angle C$ is not congruent to any angle in ΔDEF. Angles are not congruent.

23. yes; ΔJMN~ ΔJLK

24. yes; ΔABC ~ Δ EDC

25. yes; ΔVWX~ ΔVYZ

26. yes; ΔPQR ~ ΔPST

39. 14 | 40. $\dfrac{28}{11}$ | 41. 27 | 42. 36

43. 100 | 44. 13.5 | 45. 12