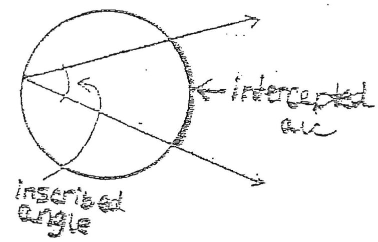
10.3 – Inscribed Angles

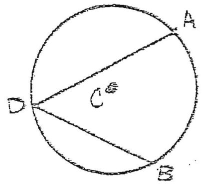
1. **Inscribed Angle** – an angle whose vertex is on a circle and whose sides contain chords of the circle.
2. **Intercepted Arc** – the arc that lies in the interior of an inscribed angle and has endpoints on the angle.



Theorem 10.8 – Measure of an Inscribed Angle

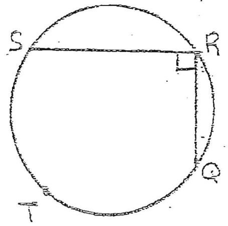
If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

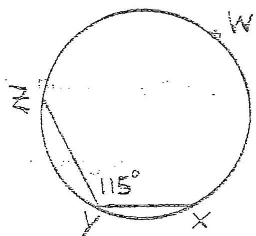
$m\angle ADB = \frac{1}{2}\, m\overset{\frown}{AB}$

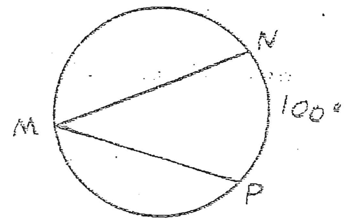


Example 1: **Find the value of each of the following.**

a) $m\overset{\frown}{QTS} =$ b) $m\overset{\frown}{ZWX} =$ c) $m\angle NMP =$

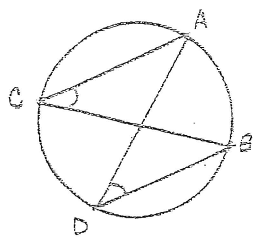






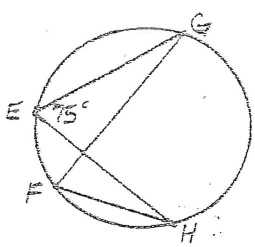
Theorem 10.9

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

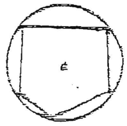


$\angle C \cong \angle D$

Example 2: Find the measure of $\angle F$.



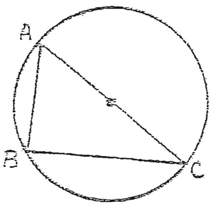
If all of the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon.



Theorem 10.10

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

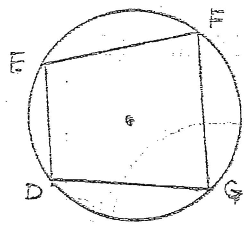
$\angle B$ is a right angle if and only if $\overline{AC}$ is a diameter of the circle.



Theorem 10.11

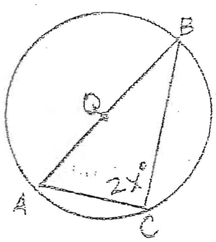
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on some circle C, if and only if $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$.

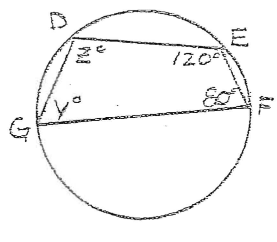


Example 3: **Find the value of each variable.**

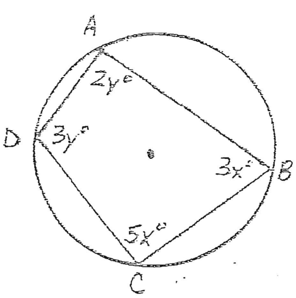
a)



b)



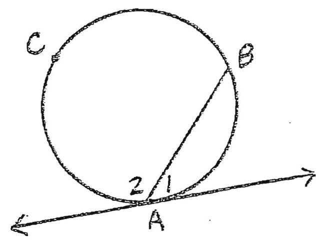
Example 4: **In the diagram, ABCD is inscribed in circle P. Find the measure of each angle.**



Theorem 10.12

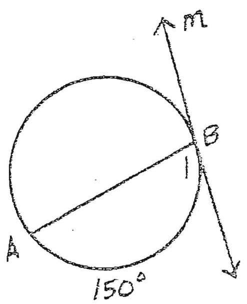
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\qquad$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$

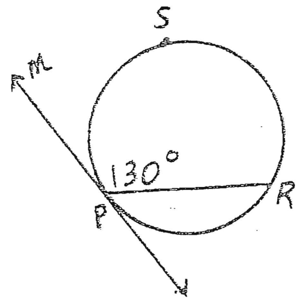


Example 1: Line m is tangent to the circle.

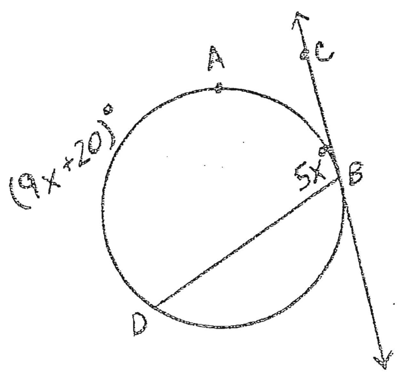
a) Find $m\angle 1$.



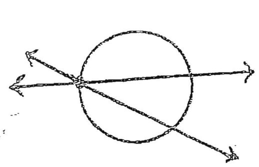
b) Find $m\overset{\frown}{RSP}$.



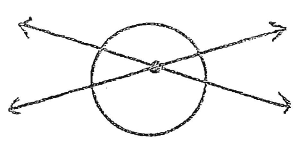
Example 2: $\overleftrightarrow{BC}$ is tangent to the circle. Find $m\angle CBD$.



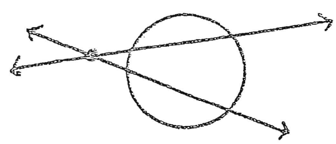
If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



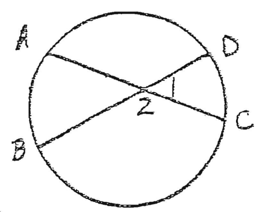
inside the circle



outside the circle

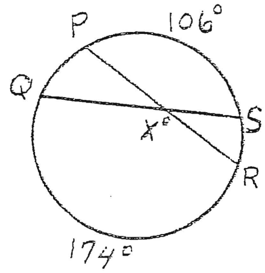
Theorem 10.13

If two chords intersect in the **interior** of a circle, then the measure of each angle is one half the **sum** of the measures of the arcs intercepted by the angle and its vertical angle.



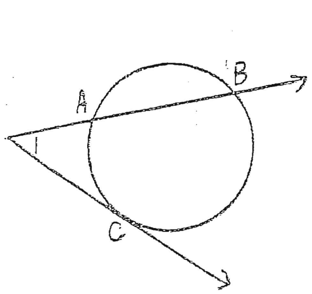
$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB}) \qquad m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$$

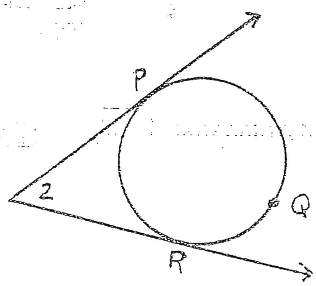
Example 3: Find the value of x.

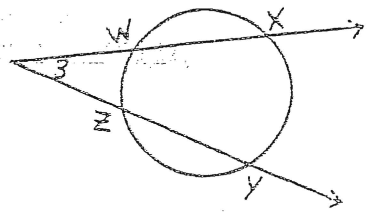


Theorem 10.14

If a tangent and a secant, two tangents, or two secants intersect in the **exterior** of a circle, then the measure of the angle formed is one half the **difference** of the measures of the intercepted arcs.



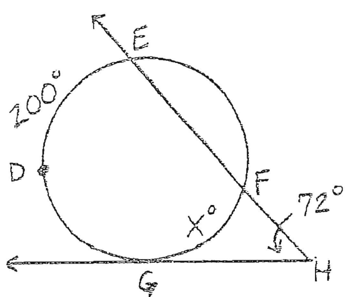




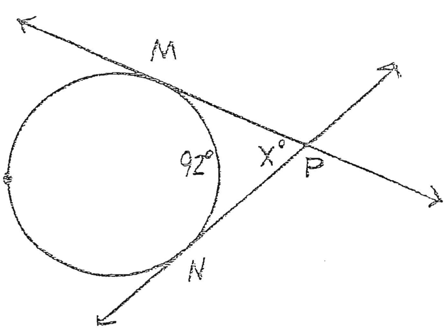
$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC}) \qquad m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR}) \qquad m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$$

Example 4: Find the value of x.

a)



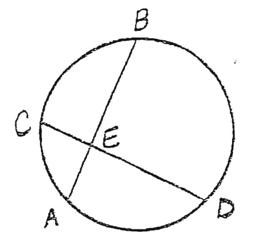
b)



10.5 – Segment Lengths in Circles

Theorem 10.15

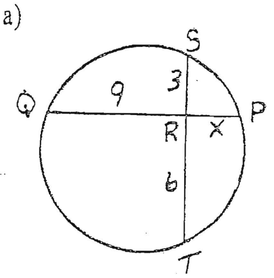
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



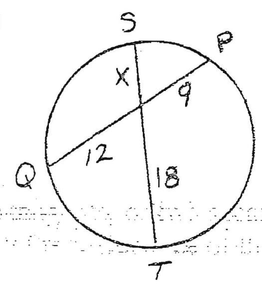
$$EA \cdot EB = EC \cdot ED$$

Example 1: Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.

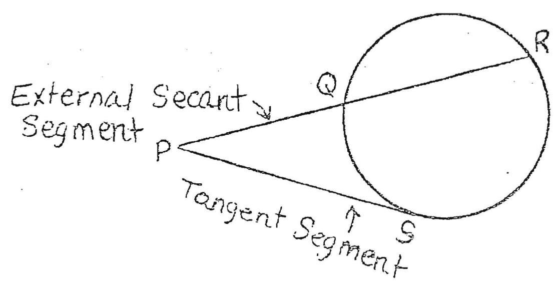
a)



b)

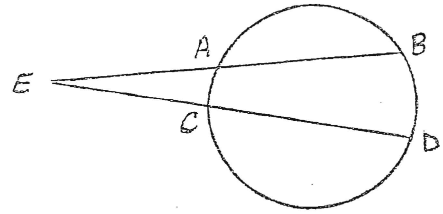


In the figure below, $\overline{PS}$ is called a **tangent segment** because it is tangent to the circle at an endpoint. Similarly, $\overline{PR}$ is a **secant segment** and $\overline{PQ}$ is the **external segment** of $\overline{PR}$.



Theorem 10.16

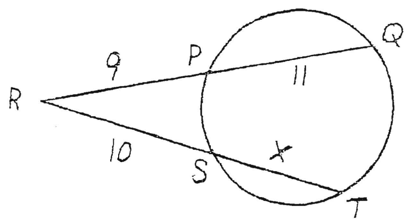
If two secant segments share the same endpoint outside the circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



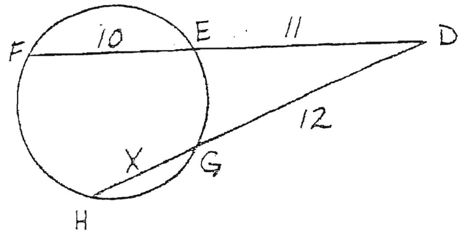
$$EA \cdot EB = EC \cdot ED$$

Example 2: Find the value of x.

a)

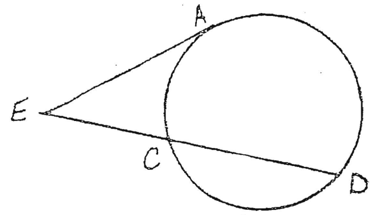


b)



Theorem 10.17

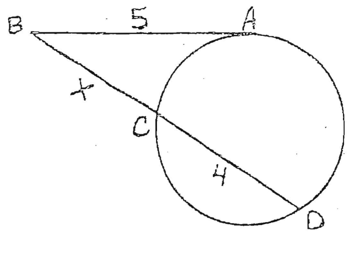
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.



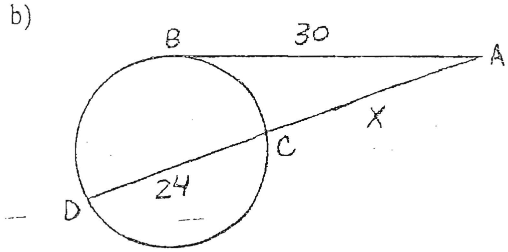
$$(EA)^2 = EC \cdot ED$$

Example 4: Find the value of x.

a)



b)



<u>10.6</u> – Equations of Circles

1. Standard Equation of a Circle

$(x-h)^2+(y-k)^2=r^2$ with radius r and center (h, k).

<u>Example 1</u>: Write the standard equation of a circle, given the center and radius.

a) center = (-4, 0)
radius = 8

b) center = (2, -1)
radius = 7.1

c) center = (0, 6)
radius = 6.5

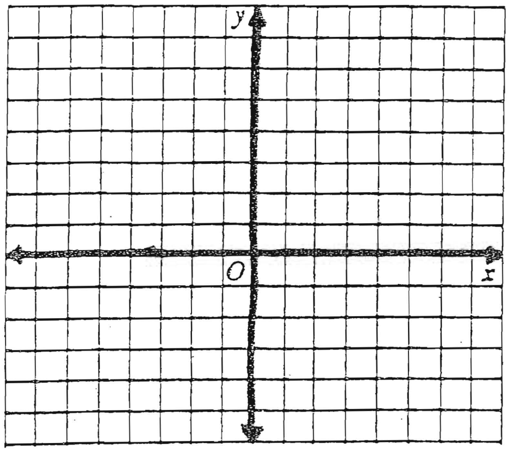
<u>Example 2</u>: Write the standard equation of a circle, given the center and a point on the circle.

a) center = (5, -1)
point on circle = (1, 2)

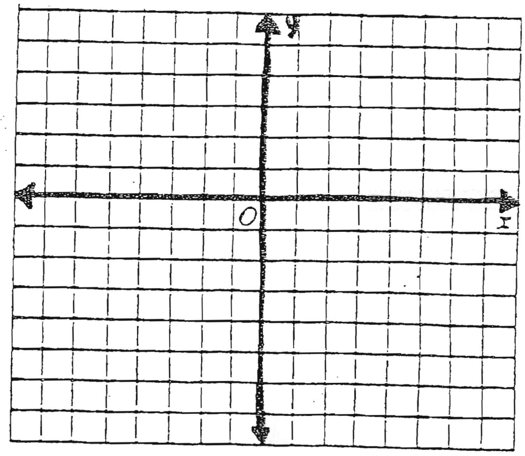
b) center = (2, 1)
point on circle = (4, -3)

Example 3: Graph each circle with the given equation.

a) $(x+2)^2+(y-3)^2=9$



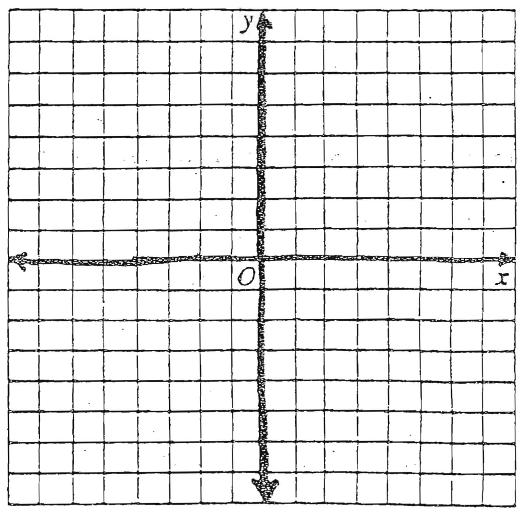
b) $(x-3)^2+(y+1)^2=4$



Example 4: Graph the circle. Plot the given points. Describe the points as outside the circle, inside the circle, or on the circle.

Circle: $(x+1)^2+y^2=16$

Points: A(5, 4) B(-5, 0) C(-2, -2)



Plug points A, B, and C into the equation of the circle. How do the results relate to their placement on/around the circle?