Geometry w/Applications Section 2.1 Conditional Statements

▶ A **conditional statement** is a logical statement that can be written in *if* p, *then* q *form*, where p is the **hypothesis** and q is the **conclusion.**

Example1: *If an animal is a monkey, then it has a tail.*

Hypothesis: an animal is a monkey
Conclusion: it has a tail

Example 2: All sharks have boneless skeleton.

If – then form: ______________________________

Hypothesis: ______________________________

Conclusion: ______________________________

▶ The **converse** of a conditional statement is formed by switching the *if* and the *then.* (If q , then p)

▶ The **inverse** is a conditional statement formed by negating the *if* and the *then.* (If not p, then not q)

▶ The **contrapositive** is a conditional statement formed by switching and negating the *if* and the *then.*
(If not q, then not p)

Example 3:

Original : If you are in Hawaii, then you are in the tropics.

Inverse: ______________________________

Converse: ______________________________

Contrapositive: ______________________________

When two statements are both true or both false, they are called **equivalent statements.**

Section 2.2 Biconditionals

▶ A **biconditional statement** is a statement that contains the phrase "*if and only if.*"

Example: A ceiling fan runs *if and only if* the light switch is on.

Point, Line and Plane Postulates

Postulate 5: Through any two points there exists exactly one line.

Postulate 6: A line contains at least 2 points.

Postulate 7: If two lines intersect, then their intersection is exactly one point.

Postulate 8: Through any 3 noncollinear points there exists exactly one plane.

Postulate 9: A plane contains at least 3 noncollinear points.

Postulate 10: If two points lie in a plane, then the line containing them lies in the plane.

Postulate 11: If two planes intersect, then their intersection is a line.

Example: Decide whether each statement is *true* or *false*. If it is false, give a counterexample.

a) A line can be in more than one plane.

b) Four noncollinear points are always coplanar.

c) Through any three points, there exists exactly one line.

Section 2.3 **Deductive Reasoning**

Conditional statements can be written using symbolic notation.

Symbolic Notation:

$p \rightarrow q$ means "*p implies q*," *or* equivalently "*if p, then q.*"

$\sim p$ means "*not p*"

$\sim q$ means "*not q*"

The converse can be written as: $q \rightarrow p$.

The inverse can be written as : $\sim p \rightarrow \sim q$

The contrapositive can be written as $\sim q \rightarrow \sim p$

A biconditional statement can be written as : $p \leftrightarrow q$ means "*p if and only if q.*"

Example1. Let p be " today is Monday" and q be " there is school today."

Write $q \rightarrow p$ in words.

____

Write $\sim q$ in words. ______________________________

Example 2: Given : If a number is divisible by 6, then it is divisible by 3.

a. Write the inverse symbolically: ________________
b. Write the inverse in words.

____

Laws of deductive reasoning .

1. Law of Detachment

If $p \rightarrow q$ is a true conditional statement and p is true , then q is true.

A valid argument

Steph knows that if she does do her chores, she will be allowed to go to the mall. Steph did her chores. Conclusion: So Steph went to the mall.

2. Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.

A valid argument.

Given: If Jess goes to the music store, she shops for a CD.

If Jess shops for a CD, then she will buy a CD.

Conclusion: (*If Jess goes to the music store, then she will buy a CD.)*________________

Jess goes to the music store. *Therefore, she bought a CD.*________________

▶ Algebraic Properties of Equality

Algebraic Properties of Equality
Let a, b, and c be real numbers.

Addition Property If $a = b$, then $a + c = b + c$.

Subtraction Property If $a = b$, then $a - c = b - c$.

Multiplication Property If $a = b$, then $ac = bc$.

Division Property If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Reflexive Property For any real number a, $a = a$.

Symmetric Property If $a = b$, then $b = a$.

Transitive Property If $a = b$ and $b = c$, then $a = c$.

Substitution Property If $a = b$, then a can be substituted for b in any equation or expression.

▶ Additional Properties.

Distributive Property	$a(b + c) = ab + ac$
Commutative Property	$a + b = b + a$
Associative Property	$a + (b + c) = (a + b) + c$

Example 1 Write a reason for each step.

$3x+17=8x+2x-18$ Given

$3x+17=10x-18$

$17=7x-18$

$35=7x$

$5=x$

Simplifying may be used as a reason for an argument

The algebraic properties of equality can be used in Geometry when justifying segment and angle relationships.

	SEGMENT LENGTH	ANGLE MEASURE
REFLEXIVE	For any segment AB, $AB = AB$.	For any angle A, $m\angle A = m\angle A$.
SYMMETRIC	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
TRANSITIVE	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Example 2 ; In the diagram AC = BD. Verify that AB = CD

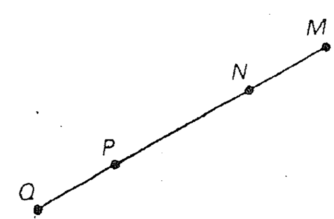
AC = BD	Given
AC = AB + BC BD = BC + CD	______________________
AB + BC = BC + CD	______________________
AB = CD	______________________

Geometry with Applications—Section 2.4

STATEMENTS	REASONS
1. $6x - 2 = -4(x - 1)$	1.____________________
2. $6x - 2 = -4x + 4$	2.____________________
3. $10x - 2 = 4$	3.____________________
4. $10x = 6$	4.____________________
5. $x = \frac{3}{5}$	5.____________________

STATEMENTS	REASONS
1. $4 + 2(3x + 5) = 11 - x$	1.____________________
2. $4 + 6x + 10 = 11 - x$	2.____________________
3. $14 + 6x = 11 - x$	3.____________________
4. $14 + 7x = 11$	4.____________________
5. $7x = -3$	5.____________________
6. $x = \frac{-3}{7}$	6.____________________

Given than MN=PQ. Show that MP=NQ



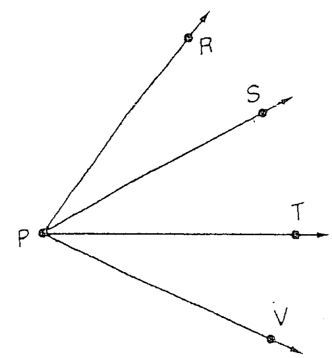
STATEMENTS	REASONS
1. MN = PQ	1.____________________
2. PN = PN	2.____________________
3. MN + PN = QP + PN	3.____________________
4. MN + PN = MP	4.____________________
5 QP + PN = QN	5.____________________
6. MP = NQ	6.____________________

Given that WY=XZ. Show that WX=YZ

STATEMENTS | REASONS

1. WY = XZ — 1. ______________________
2. WY = WX + XY — 2. ______________________
3. XZ = XY + ~~X~~YZ — 3. ______________________
4. WX + XY = XY + YZ — 4. ______________________
5. WX = YZ — 5. ______________________

Given that $m\angle RPS = m\angle TPV$; $m\angle TPV = m\angle SPT$
Show that $m\angle RPV = 3(m\angle RPS)$



STATEMENTS | REASONS

1. $m\angle RPS = m\angle TPV$
   $m\angle TPV = m\angle SPT$ — 1. ______________________
2. $m\angle RPS = m\angle SPT$ — 2. ______________________
3. $m\angle RPV = m\angle RPT + m\angle TPV$ — 3. ______________________
4. $m\angle RPT = m\angle RPS + m\angle SPT$ — 4. ______________________
5. $m\angle RPV = m\angle RPS + m\angle SPT + m\angle TPV$ — 5. ______________________
6. $m\angle RPV = m\angle RPS + m\angle RPS + m\angle RPS$ — 6. ______________________
7. $m\angle RPV = 3(m\angle RPS)$ — 7. ______________________

Section 2.5 Proving Statements about Segments

<u>Theorem</u> is a statement that follows as a result of other true statements. All theorems must be proved.

THEOREM 2.1 *Properties of Segment Congruence*

Segment congruence is reflexive, symmetric, and transitive. Here are some examples:

REFLEXIVE	For any segment AB, $\overline{AB} \cong \overline{AB}$.
SYMMETRIC	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
TRANSITIVE	If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Recall the definition of Congruency. If $\overline{AB} \cong \overline{BC}$ then $AB = BC$.

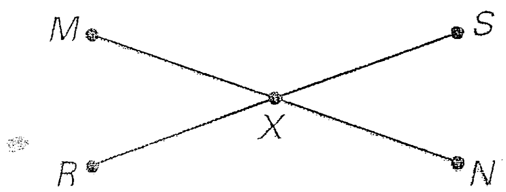
Example 1. Use the diagram and the given information to complete each reason.

Given: EF = GH

Prove : $\overline{EG} \cong \overline{FH}$

Statements	Reasons
1. EF = GH	________________
2. EF + FG = GH + FG	________________
3. EF + FG = EG GH + FG = FH	________________
4. EG = FH	________________
5. $\overline{EG} \cong \overline{FH}$	________________

1.

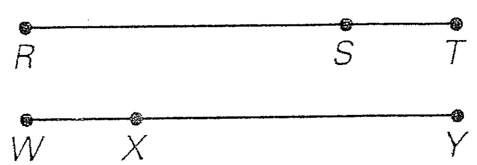


Given: X is the midpoint of $\overline{MN}$, and $MX = RX$.

Prove: $XN = RX$

Statement	Reason
1. X is the midpoint of $\overline{MN}$	1.____________________
2. XN = MX	2.____________________
3. MX = RX	3.____________________
4. XN = RX	4.____________________

2.



Given: $\overline{RT} \cong \overline{WY}$, $ST = WX$

Prove: $\overline{RS} \cong \overline{XY}$

1. $\overline{RT} \cong \overline{WY}$	1.____________________
2. RT = WY	2.____________________
3. RT = RS + ST WY = WX + XY	3.____________________
4. RS + ST = WX + XY	4.____________________
5. ST = WX	5.____________________
6. RS = XY	6.____________________
7. $\overline{RS} \cong \overline{XY}$	7.____________________

Section 2.6 <u>Proving Statements about Angles</u>

THEOREM 2.2 ***Properties of Angle Congruence***

Angle congruence is reflexive, symmetric, and transitive.

Here are some examples.

REFLEXIVE	For any angle A, $\angle A \cong \angle A$.
SYMMETRIC	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
TRANSITIVE	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

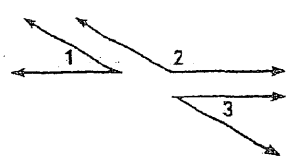
THEOREM 2.3 ***Right Angle Congruence Theorem***

All right angles are congruent.

THEOREM 2.4 ***Congruent Supplements Theorem***

If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.

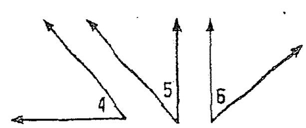
If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$.



THEOREM 2.5 ***Congruent Complements Theorem***

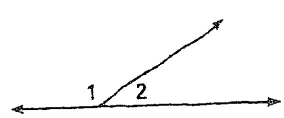
If two angles are complementary to the same angle (or to congruent angles) then the two angles are congruent.

If $m\angle 4 + m\angle 5 = 90°$ and $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.



POSTULATE 12 ***Linear Pair Postulate***

If two angles form a linear pair, then they are supplementary.



$m\angle 1 + m\angle 2 = 180°$

ple 1 In the diagram, $m\angle 8 = m\angle 5$ and $m\angle 5 = 125°$.