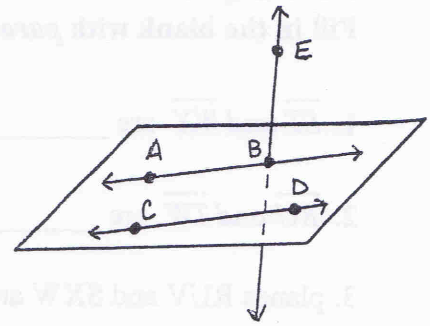
Chapter 3: Perpendicular and Parallel Lines

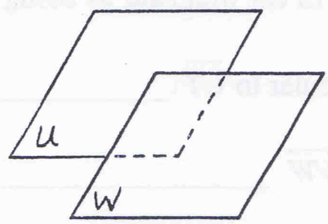
3.1 – Lines and Angles



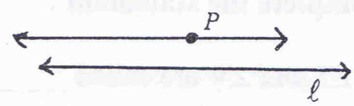
1. Parallel Lines – lines that are coplanar and do not intersect.
   ($\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$)

2. Skew Lines – lines that do not intersect and are not coplanar.
   ($\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$)

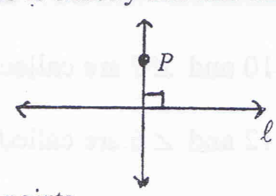
3. Parallel Planes – planes that do not intersect.
   (planes *U* and *W*)



4. Postulate 13 – Parallel Postulate
   If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.
   (There is exactly one line through *P*, parallel to ℓ)



5. Postulate 14 – Perpendicular Postulate
   If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.
   (There is exactly one line through *P*, perpendicular to ℓ)



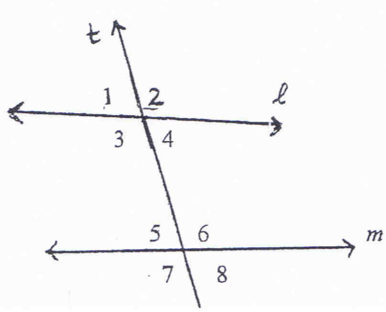
6. Transversal – a line that intersects two or more coplanar lines at different points.
   ('t' in the diagram below)

7. Corresponding Angles – two angles that occupy corresponding positions.
   (angles 1 and 5 in the diagram below)

8. Alternate Exterior Angles – two angles that lie outside the two lines on opposite sides of the transversal.
   (angles 1 and 8 in the diagram below)

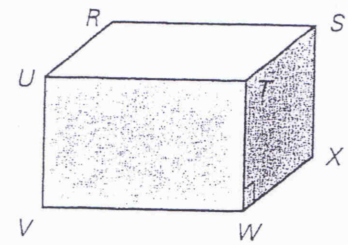
9. Alternate Interior Angles – two angles that lie between the two lines on opposite sides of the transversal.
   (angles 3 and 6 in the diagram below)

10. Consecutive Interior Angles – (Same Side Interior Angles) two angles that lie between the two lines on the same side of the transversal. (angles 3 and 5 in the diagram below)



3.1 Examples

Fill in the blank with *parallel*, *skew*, or *perpendicular*.



1. $\overleftrightarrow{ST}$ *and* $\overleftrightarrow{WX}$ are ______________________________

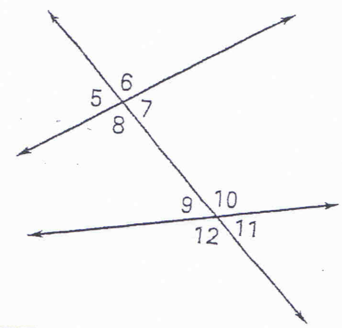
2. $\overleftrightarrow{RU}$ *and* $\overleftrightarrow{TW}$ are ______________________________

3. planes RUV and SXW are ______________________

Think of each segment in the diagram as being part of a line.

4. Name a line perpendicular to $\overleftrightarrow{UT}$. ______________________

5. Name a line skew to $\overleftrightarrow{VW}$ _________________________



Complete the statement .

6. $\angle 5$ *and* $\angle 9$ are called ______________________________________

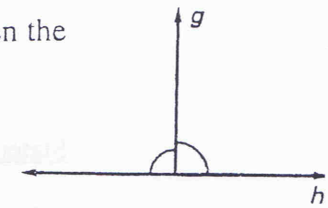
7. $\angle 8$ *and* $\angle 10$ are called ____

8. $\angle 10$ and $\angle 7$ are called ____

9. $\angle 12$ and $\angle 6$ are called ____

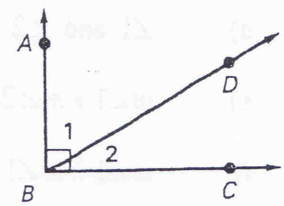
Section 3.2 –**Proof and Perpendicular Lines**

Theorem 3.1 If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.



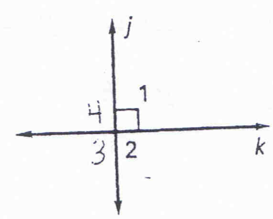
$g \perp h$

Theorem 3.2 If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.



$\angle 1$ and $\angle 2$ are complementary

Theorem 3.3 If two lines are perpendicular, then they intersect to form **4** right angles.

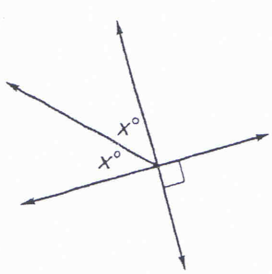


$\angle 2$ is a right angle.

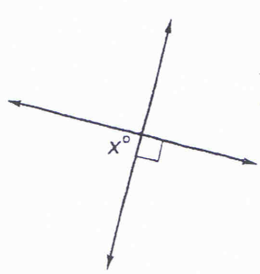
Exercises

Find the value of x.

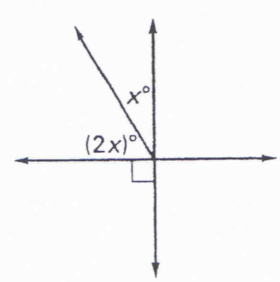
1.



2.

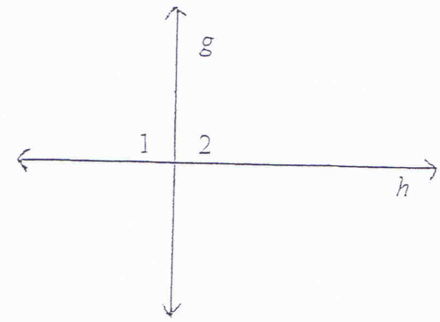


3.



Ex: *Given:* $\angle 1 \cong \angle 2$
$\angle 1$ and $\angle 2$ are a linear pair.

Prove: $g \perp h$



Statements		Reasons
a)	$\angle 1 \cong \angle 2$	a) ________
b)	$m\angle 1 = m\angle 2$	b) ________
c)	$\angle 1$ and $\angle 2$ are a linear pair.	c) ________
d)	$\angle 1$ and $\angle 2$ are supplementary	d) ________
e)	$m\angle 1 + m\angle 2 = 180°$	e) ________
f)	$m\angle 1 + m\angle 1 = 180°$	f) ________
g)	$2(m\angle 1) = 180°$	g) ________
h)	$m\angle 1 = 90°$	h) ________
i)	$\angle 1$ is a right angle	i) ________
j)	$g \perp h$	j) ________

3.3 – Parallel Lines and Transversals

1. **Postulate 15 – Corresponding Angles Postulate**

   If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

   If $l \parallel m$, then $\angle 1 \cong \angle 5$.

2. **Theorem 3.4 – Alternate Interior Angles Theorem**

   If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

   If $l \parallel m$, then $\angle 4 \cong \angle 5$.

3. **Theorem 3.5 – Consecutive Interior Angles Theorem**

   If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

   If $l \parallel m$, then $m\angle 3 + m\angle 5 = 180°$.

4. **Theorem 3.6 – Alternate Exterior Angles Theorem**

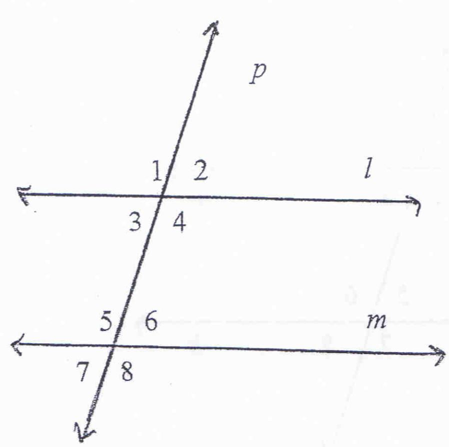
   If parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

   If $l \parallel m$, then $m\angle 2 \cong \angle 7$.

5. **Theorem 3.7 – Perpendicular Transversal Theorem**

   If a transversal is perpendicular to one of the two parallel lines, then it is perpendicular to the other.

   If $l \parallel m$ and $p \perp l$, then $p \perp m$.



3.4 – Proving Lines are Parallel

1. **Postulate 16 – Corresponding Angles Converse**

   If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

   If $\angle 3 \cong \angle 7$, then $j \parallel k$.

2. **Theorem 3.8 – Alternate Interior Angles Converse**

   If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.

   If $\angle 3 \cong \angle 6$, then $j \parallel k$.

3. **Theorem 3.9 – Consecutive Interior Angles Converse**

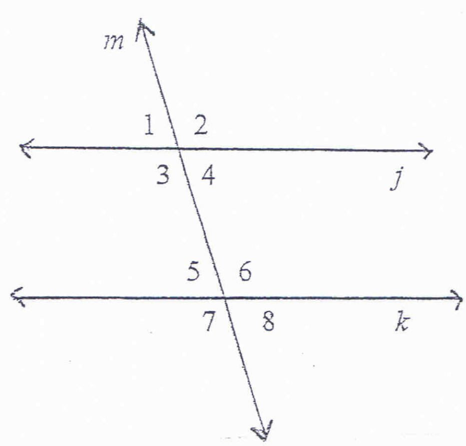
   If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.

   If $\angle 4 + \angle 6 = 180°$, then $j \parallel k$.

4. **Theorem 3.10 – Alternate Exterior Angles Converse**

   If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.

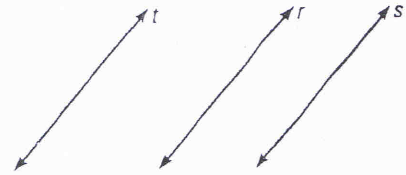
   If $\angle 1 \cong \angle 8$, then $j \parallel k$.



3.5-Using Properties of Parallel Lines

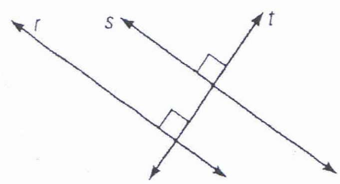
Theorem 3.11
If two lines are parallel to the same line, then they are parallel to each other.

. **GIVEN** $r \parallel t, t \parallel s$



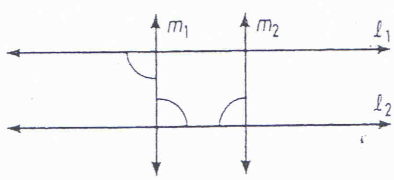
Theorem 3.12
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

. **GIVEN** $r \perp t, t \perp s$

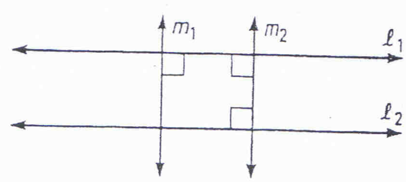


Naming parallel lines.
Determine which lines, if any, are parallel

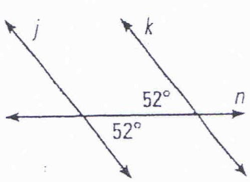
1.



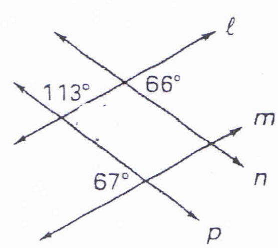
2.



3.



4.



Parallel and Perpendicular Lines in the Coordinate Plane
Intro to 3.6 and 3.7

Slope – The ratio of the vertical change (the rise) to the horizontal change (the run).

If a line passes through the points (x_1, y_1) and (x_2, y_2), then the slope is given by:

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Exercises – Find the slope of the line passing through the following points.

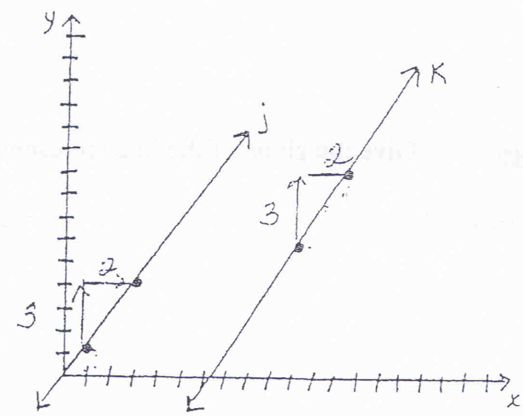
1. (3, -4)
   (-1, 5)

2. (0, -8)
   (-1, -1)

3. (5, 2)
   (7, 9)

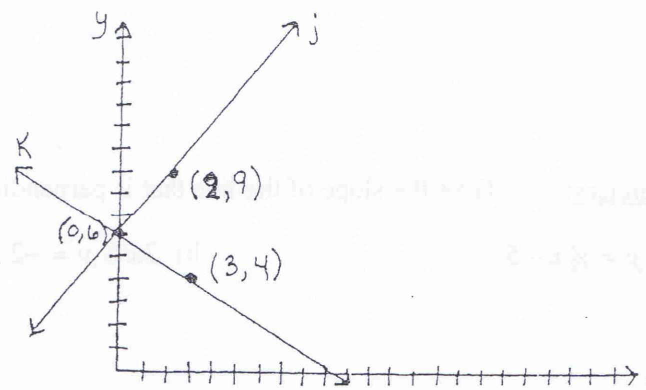
Postulate 17 – Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.



Postulate 18 – Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1 (the slopes are opposite reciprocals). Vertical and horizontal lines are perpendicular to each other.



Exercises: Fill in the chart based on the previous postulates.

Given a line with slope:	A line parallel to the original line would have a slope of:	A line perpendicular to the original line would have a slope of:
$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{3}{1}$
-2		
$\frac{5}{4}$		
0		
$-\frac{2}{3}$		
1		
$\frac{1}{4}$		

4. Slope Intercept Form –

$$y = mx + b$$

where m = slope and b = y-intercept.

Exercises: Give the slope of the line represented by the following equations.

a) $y = -3x + 4$ b) $3x = 2y + 1$ c) $4y = 2x + 8$

Exercises: Give the slope of the line that is parallel to the following equations.

a) $y = -6x - 1$ b) $-4y = 3x - 6$ c) $2x = y + 7$

Exercises: Give the slope of the line that is perpendicular to the following equations.

a) $y = \frac{1}{2}x - 5$ b) $3x + y = -2$ c) $-6y + 6x = 12$