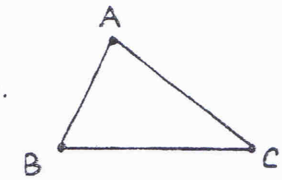
Chapter 4 – Congruent Triangles

4.1 – Triangles and Angles

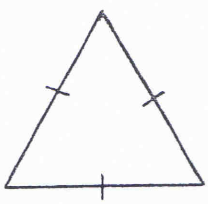


1. Triangle – a figure formed by three segments joining three noncollinear points.
2. Vertex – (plural *vertices*) Each of the three points joining the sides of a triangle.
3. Adjacent Sides – two sides sharing a common vertex.

Names of Triangles

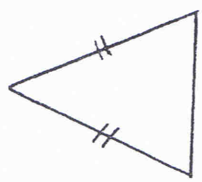
Classification by Sides:

Equilateral Triangle



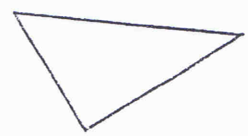
3 congruent sides

Isosceles Triangle



at least 2 congruent sides

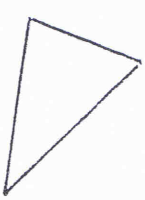
Scalene Triangle



no congruent sides

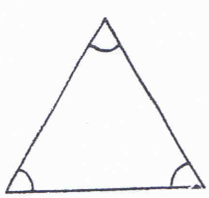
Classification by Angles:

Acute Triangle



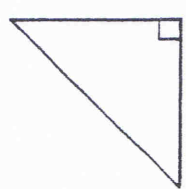
3 acute angles

Equiangular Triangle



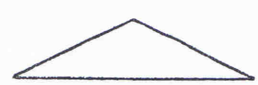
3 congruent angles

Right Triangle



1 right angle

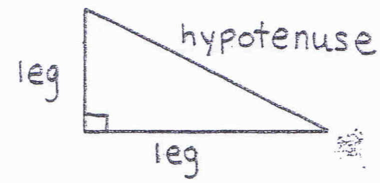
Obtuse Triangle



1 obtuse angle

4.1 (cont.)

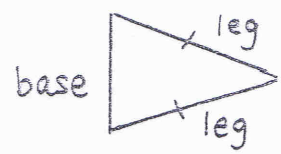
Right Triangles:



4. Legs – the sides that form the right angle.

5. Hypotenuse – the side opposite the right angle.

Isosceles Triangles:



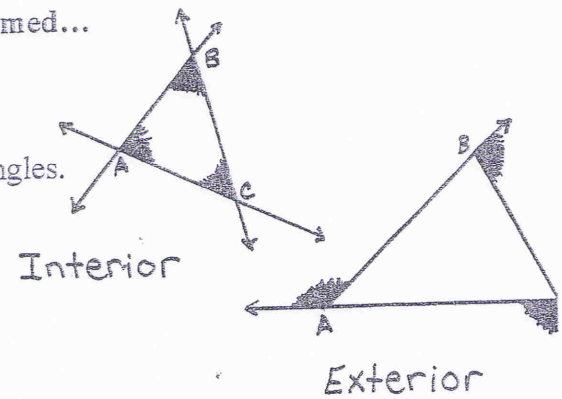
6. Legs – the two congruent sides.

7. Base – the third side (not congruent to either of the other sides).

When the sides of a triangle are extended, other angles are formed...

8. Interior Angles – the three original angles.

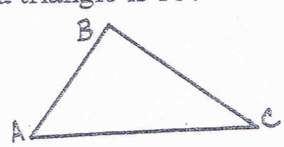
9. Exterior Angles – the angles that are adjacent to the interior angles.



Theorem 4.1 - Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

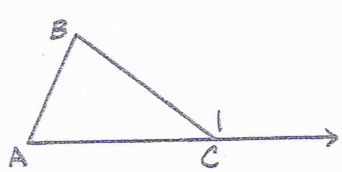
$m\angle A + m\angle B + m\angle C = 180^\circ$



Theorem 4.2 – Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$m\angle 1 = m\angle A + m\angle B$

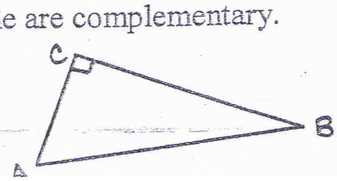


10. Corollary to a Theorem – a statement that can be proved easily using the theorem.

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

$m\angle A + m\angle B = 90^\circ$

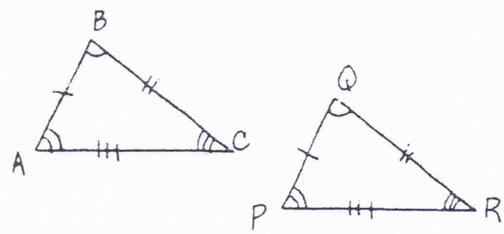


4.2 – Congruence and Triangles

1. When two figures are **congruent**, there is a correspondence between their angles and sides such that **corresponding angles** are congruent and **corresponding sides** are congruent.

Ex For the triangles below, you can write $\triangle ABC \cong \triangle PQR$, which is read "triangle ABC is congruent to triangle PQR."

Corresponding Angles	**Corresponding Sides**
$\angle A \cong \angle P$	$\overline{AB} \cong \overline{PQ}$
$\angle B \cong \angle Q$	$\overline{BC} \cong \overline{QR}$
$\angle C \cong \angle R$	$\overline{CA} \cong \overline{RP}$

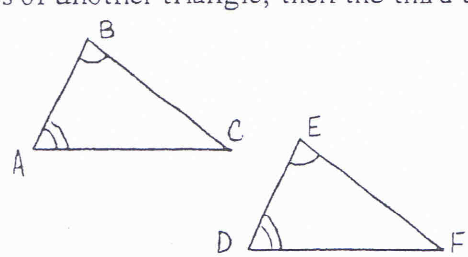


- There is more than one way to write a congruence statement, but it is important to list the corresponding angles in the same order. You could write $\triangle BCA \cong \triangle QRP$, but you can not write $\triangle CBA \cong \triangle PQR$.

2. **Theorem 4.3 – Third Angles Theorem**

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
then $\angle C \cong \angle F$.

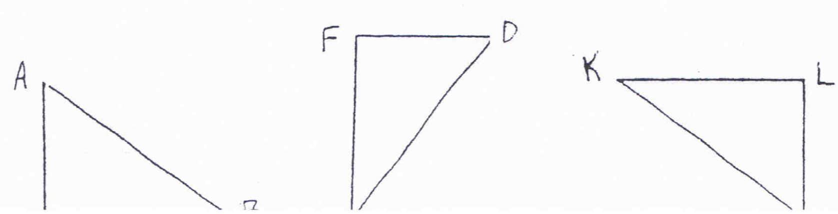


3. **Theorem 4.4 – Properties of Congruent Triangles**

Reflexive Property of Congruent Triangles – Every triangle is congruent to itself.

Symmetric Property of Congruent Triangles – If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

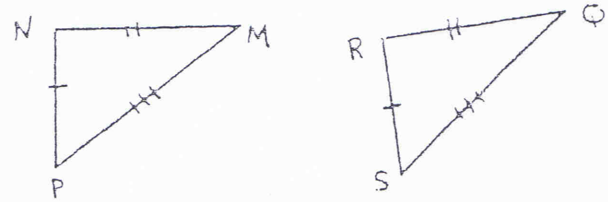
Transitive Property of Congruent Triangles – If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.3 – Proving Triangles are Congruent: SSS and SAS

1. **Postulate 19 – Side-Side-Side Congruence Postulate (SSS)**

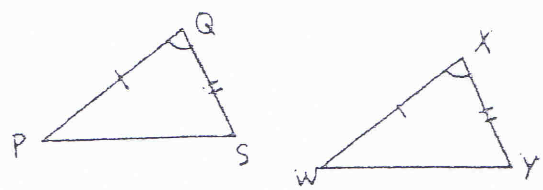
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.



$\triangle MNP \cong \triangle QRS$

2. **Postulate 20 – Side-Angle-Side Congruence Postulate (SAS)**

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

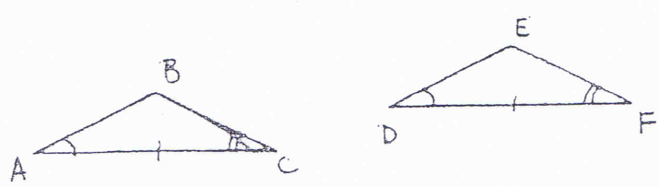


$\triangle PQS \cong \triangle WXY$

4.4 – Proving Triangles are Congruent: ASA and AAS

3. **Postulate 21 – Angle-Side-Angle Congruence Postulate (ASA)**

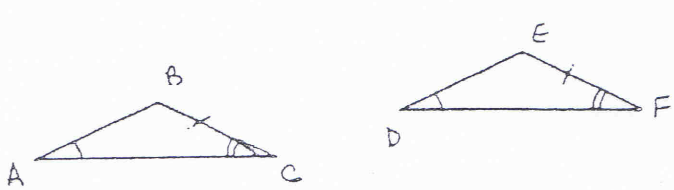
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.



$\triangle ABC \cong \triangle DEF$

4. **Theorem 4.5 – Angle-Angle-Side Congruence Theorem (AAS)**

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.



$\triangle ABC \cong \triangle DEF$

4.5 Using Congruent Triangles

A useful technique for showing two segments or two angles are congruent is to show that they are parts of congruent triangles.
Recall that SSS, ASA, SAS, and AAS can be used to prove the triangles congruent.

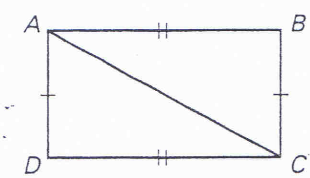
Corresponding parts of congruent triangles are congruent. **CPCTC**

Using CPCTC we can conclude that additional corresponding parts (other than those used to prove the triangles are congruent) are also congruent.

We can further deduce that lines are parallel or perpendicular.

Given: $\overline{AD} \cong \overline{BC}$
$\overline{AB} \cong \overline{DC}$

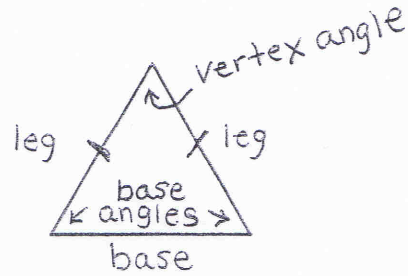
Prove: $\overline{AD} \parallel \overline{BC}$



1. $\overline{AD} \cong \overline{BC}$ — 1.____________________
2. $\overline{AB} \cong \overline{DC}$ — 2.____________________
3. $\overline{AC} \cong \overline{AC}$ — 3.____________________
4. $\triangle ADC \cong \triangle CBA$ — 4.____________________
5. $\angle DAC \cong \angle BCA$ — 5.____________________
6. $\overline{AD} \parallel \overline{BC}$ — 6.____________________

4.6 – Isosceles, Equilateral, and Right Triangles

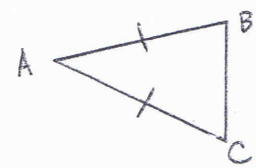
Isosceles Triangles



1. Base Angles – The two angles adjacent to the base.
2. Vertex Angle – The angle opposite the base.
3. Theorem 4.6 – Base Angles Theorem

   If two sides of a triangle are congruent, then the angles opposite them are congruent.

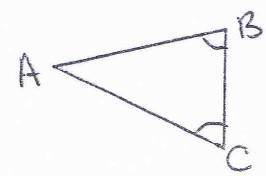
   If $\overline{AB} \cong \overline{AC}$,
   then $\angle B \cong \angle C$.



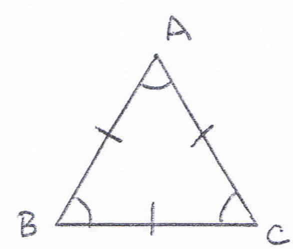
4. Theorem 4.7 – Converse of the Base Angles Theorem

   If two angles of a triangle are congruent, then the sides opposite them are congruent.

   If $\angle B \cong \angle C$,
   then $\overline{AB} \cong \overline{AC}$.



Equilateral Triangles



5. Corollary to Theorem 4.6

   If a triangle is equilateral, then it is equiangular.

6. Corollary to Theorem 4.7

   If a triangle is equiangular, then it is equilateral.

Right Triangles

7. Theorem 4.8 – Hypotenuse-Leg (HL) Congruence Theorem

   If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are congruent.

   If $\overline{BC} \cong \overline{EF}$ and
   $\overline{AC} \cong \overline{DF}$, then
   $\triangle ABC \cong \triangle DEF$