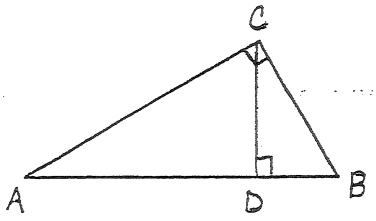
Chapter 9 – Right Triangles and Trigonometry

9.1 – Similar Right Triangles

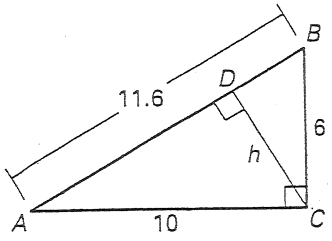
1. **Theorem 9.1** – If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$\Delta CBD \sim \Delta ABC$
$\Delta ACD \sim \Delta ABC$
$\Delta CBD \sim \Delta ACD$



Example 1 – Consider the right triangle shown.

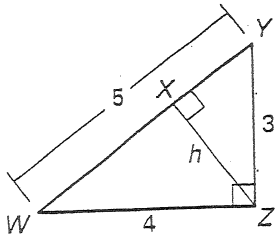
a) Identify the similar triangles.



b) Find the height, h of ΔABC.

Example 2 – Find the height, h of the given right triangle.

a)



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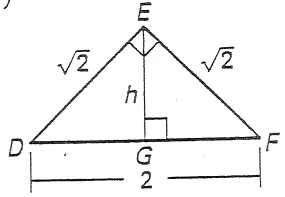

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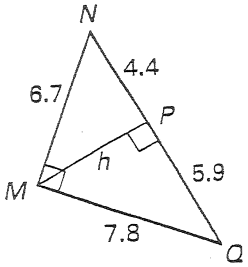
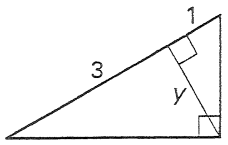


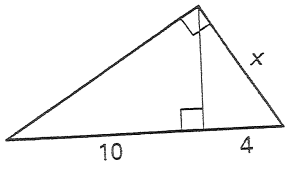
2. **Theorem 9.2** – In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.
The length of the altitude is the geometric mean of the lengths of the two segments.

Example 3 – Find the value of y.



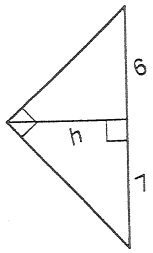
3. **Theorem 9.3** – In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Example 4 – Find the value of x.



Example 5 – Find the value of each variable.

a)



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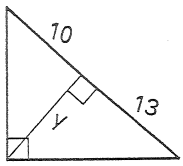

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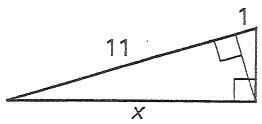
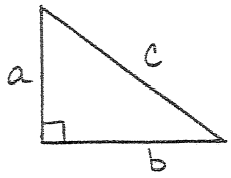


9.2 – The Pythagorean Theorem

1. Theorem 9.4 – Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



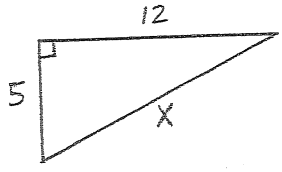
$$a^2 + b^2 = c^2$$

2. Pythagorean Triple – a set of three positive integers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$.

Example 1: 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

Example 2: Use the Pythagorean Theorem to find the missing length. Tell whether the side lengths form a Pythagorean triple.

a)



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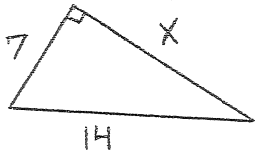

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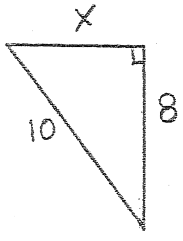
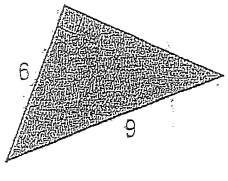


Example 3: Find the area of the triangle to the nearest tenth of a meter.

a)



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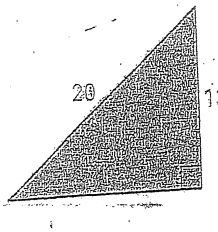

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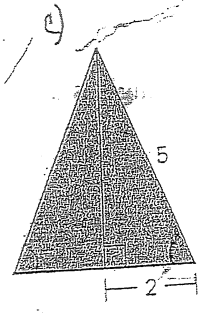


9.3 – The Converse of the Pythagorean Theorem

1. Theorem 9.4 – **Converse of the Pythagorean Theorem:**
   If $c^2 = a^2 + b^2$, then ΔABC is a right triangle.

2. Theorem 9.6 – If $c^2 < a^2 + b^2$, then ΔABC is an acute triangle.

3. Theorem 9.7 – If $c^2 > a^2 + b^2$, then ΔABC is an obtuse triangle.

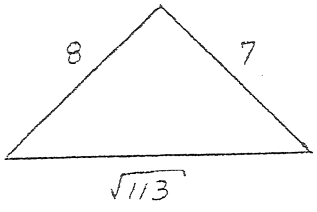
Example 1: Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as right, acute, or obtuse.

a) 2, 10, 11 b) 13, 5, 7 c) 38, 77, 86

d) 10.5, 36.5, 37.5 e) 5, 11, 6 f) 6, 8, 10

Example 2: Tell whether the triangle is a right triangle.

a)



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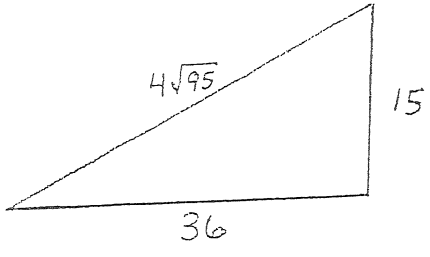
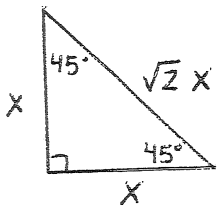


9.4 – Special Right Triangles

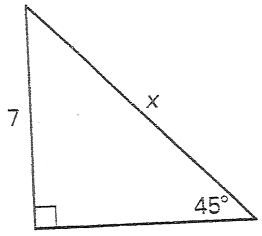
Theorem 9.8 – $45° - 45° - 90°$ Triangle Theorem

In a $45° - 45° - 90°$ triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



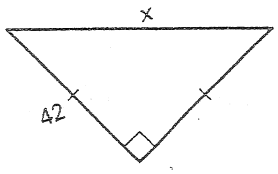
Hypotenuse = $\sqrt{2}$ (leg)

Example 1: Find the value of x.



Example 2: Find the value of each variable.

a)



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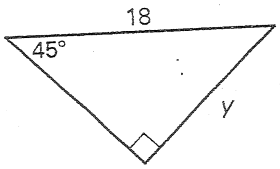

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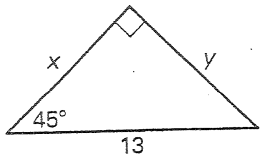
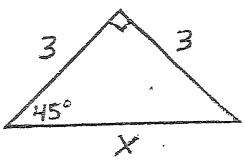
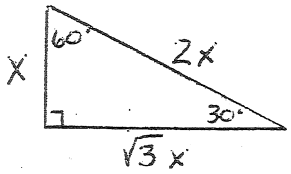


d)



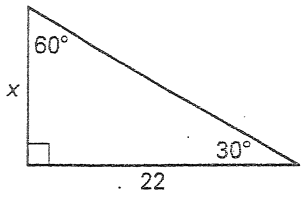
Theorem 9.9 – $30° - 60° - 90°$ **Triangle Theorem**

In a $30° - 60° - 90°$ triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



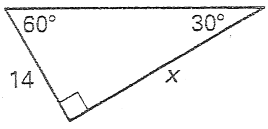
Hypotenuse = 2 (shorter leg)
Longer Leg = $\sqrt{3}$ (shorter leg)

Example 3: Find the value of x.

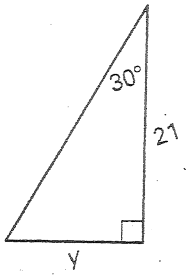


Example 4: Find the value of each variable.

a)



b)



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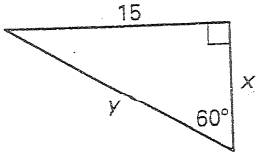
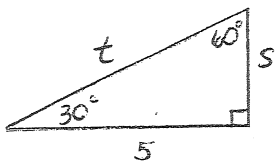


d)



9.5 – Trigonometric Ratios

1. **Trigonometric Ratio** – a ratio of the lengths of two sides of a right triangle.

2. ***Trigonometry*** – derived from the ancient Greek language and means measurement of triangles.

3. The three basic trigonometric ratios are **sine**, **cosine**, and **tangent**. (abbreviated *sin*, *cos*, and *tan*)

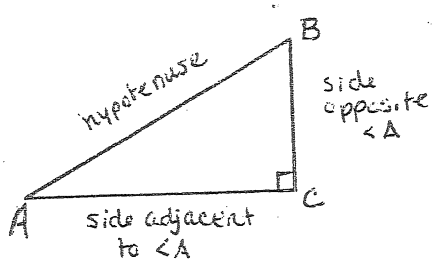
4. **Trigonometric Ratios**

Let ΔABC be a right triangle. The sine, the cosine, and the tangent of the acute angle $\angle A$ are defined as follows:

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A}$$



SOH CAH TOA **SOH CAH TOA** **SOH CAH TOA**

Example 1: Use the diagrams at the right to find the trigonometric ratio.

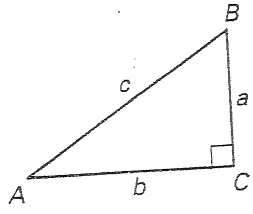
a) sin A

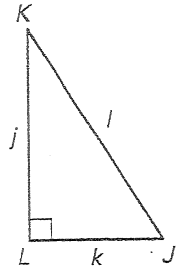
b) cos A

c) tan B

d) sin J

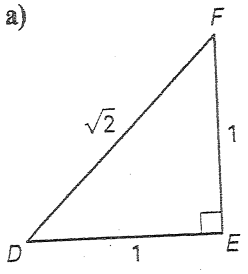
e) tan K



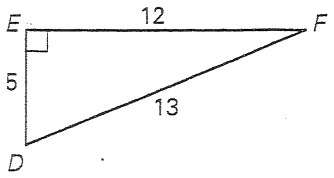


Example 2: **Find the sine, cosine, and tangent of the acute angles of the triangle. Express each value as a decimal rounded to four decimal places.**

a)



b)



Example 3: **Use a calculator to approximate the given value to four decimal places. (make sure your calculator is in degree mode)**

a) sin 30°

b) cos 18°

c) tan 72°

d) sin 48°

e) tan 42°

f) cos 65°

g) tan 14°

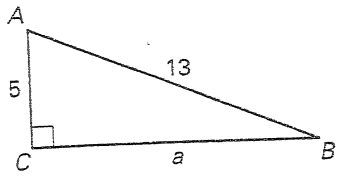
9.6 – Solving Right Triangles

1. To **solve a right triangle** means to determine the measures of all six parts (3 angles and 3 sides).

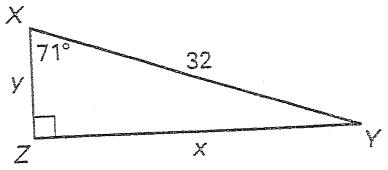
2. You can solve a right triangle is you know either of the following:

   - two side lengths
   - one side length and one acute angle measure

Example 1: Solve the right triangle. (two side lengths)

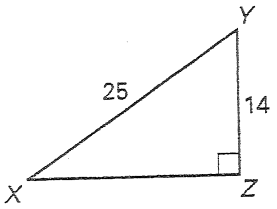


Example 2: Solve the right triangle. (one side and one acute angle)



Example 3: Solve the right triangles.

a)



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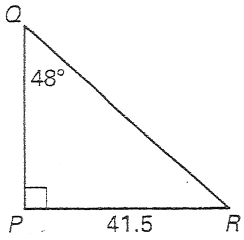

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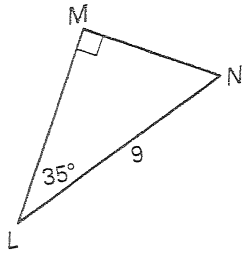
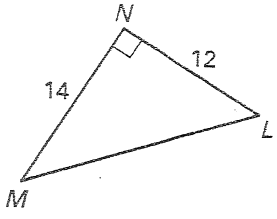
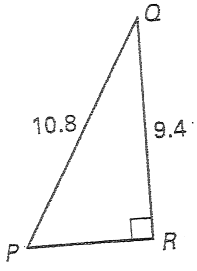


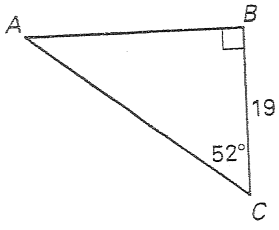
d)



e)

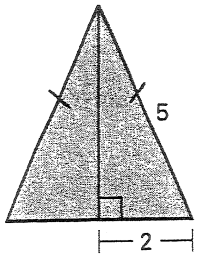


f)



4. Find the area of each triangle.

a)



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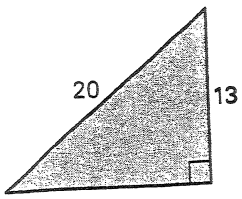
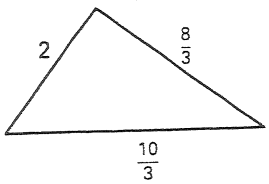
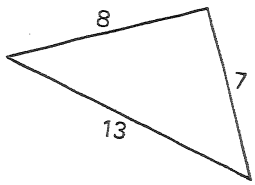


5. Tell whether each triangle is a right triangle. (yes or no)

a)



b)



6. Decide whether the numbers can represent the side lengths of a triangle.
If not, write *no*.
If so, classify the triangle as *acute*, *right*, or *obtuse*.

a) 2.7, 7.3, 10

b) 7, 7, 10

c) 3, 1.25, 3.25

d) 48, 52, 21