11.1- Angle Measures in Polygons

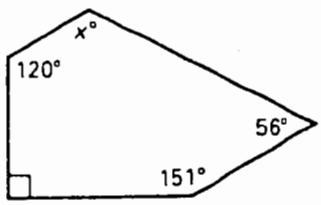
Theorem 11.1: **Polygon Interior Angles Theorem**

The sum of the measures of the interior angles of a convex *n-gon* is $(n-2)180°$.

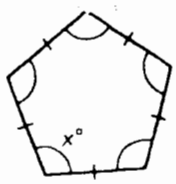
Corollary To Theorem 11.1

The measure of each interior angle of a regular *n-gon* is $\frac{(n-2)\cdot 180°}{n}$

Example 1

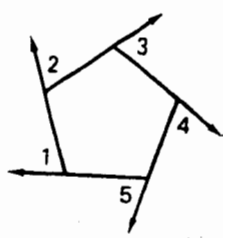


Example 2



Theorem 11.2: **Polygon Exterior Angle Theorem**

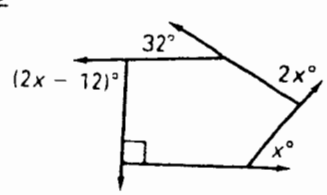
The sum of the measures of the exterior angles of a convex polygon, one at each vertex , is 360°.



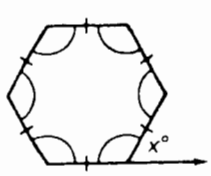
Corollary To Theorem 11.2

The measure of each exterior angle of a regular *n-gon* is $\frac{360°}{n}$.

Example 3



Example 4

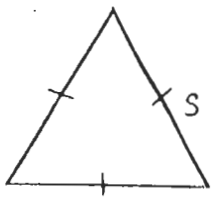


Example 5: The measure of each interior angle of a regular polygon is 165°. How many sides does the polygon have?

11.2 – Areas of Regular Polygons

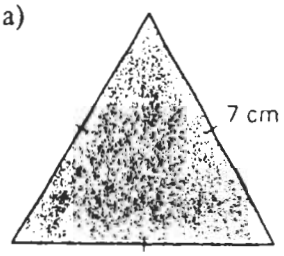
1. Theorem 11.3 – Area of an Equilateral Triangle

$$A = \frac{\sqrt{3}\, s^2}{4}$$

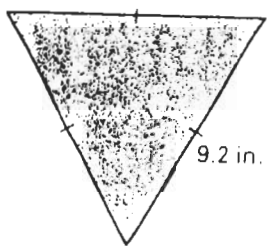


Example 1 – Find the area of each triangle.

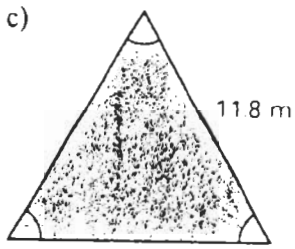
a)



b)



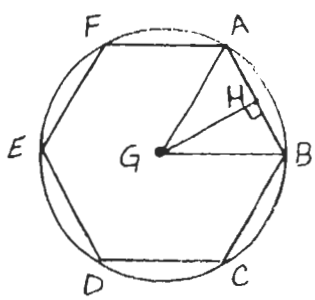
c)



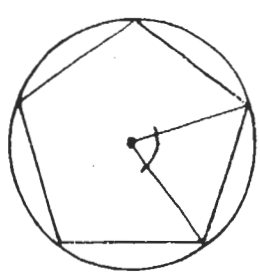
2. Center of a Regular Polygon – the center of its circumscribed circle. (Point G)

3. Radius of a Regular Polygon – the radius of its circumscribed circle ($\overline{GA}$)

4. Apothem of a Polygon – the distance from the center to any side of a regular polygon. ($\overline{GH}$)



5. Central Angle of a Regular Polygon – an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon
To find the measure of the central angle, divide 360° by the number of sides



6. Theorem 11.4 – Area of a Regular Polygon

$A = \frac{1}{2}(a)(P)$ where a = apothem and P = perimeter of the polygon

or

$A = \frac{1}{2}(a)(n)(s)$ where a = apothem, n = number of sides and s = side length

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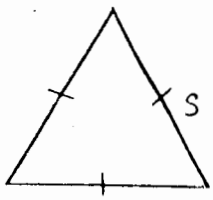

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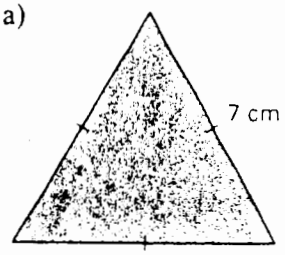

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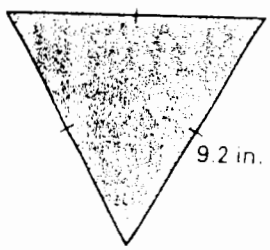

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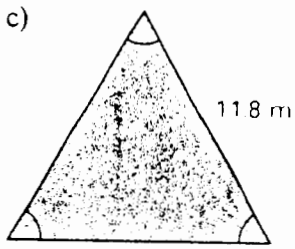

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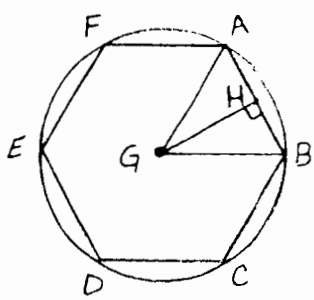

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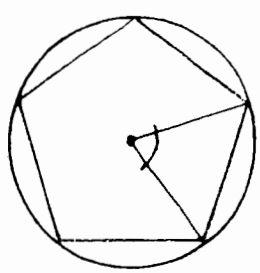

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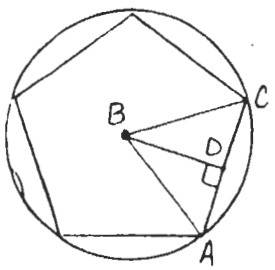
Example 2 – A regular pentagon is inscribed in a circle with radius 1 unit
Find the area of the pentagon.



1. To apply the area formula, you must find the apothem and the perimeter.
2. The measure of the central angle $\angle ABC$ = =
3. Redraw the inner triangle and use trig to find apothem BD and perimeter $5[2(DC)]$.

4. Use the area formula with the perimeter and the apothem to get the final answer.

Example 3 – A regular octagon is inscribed in a circle with radius 2 units.
Find the perimeter and the area of the octagon